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ABSTRACT

This document is the final report of the Regional Training Workshop for the Development of Materials for Mathematics Education in Asia and the Pacific, using a problem-solving approach. The workshop was committed to identifying materials which are readily available at low cost, in sufficient quantities to be used by children in classrooms in any country within the region. In the introductory chapter, the objectives and procedures of the workshop are described. Chapter 2 discusses priorities in teaching and learning. Research and the rationale for using materials, what a problem-solving approach means, key topics, and materials to be used are described. In chapter 3, lesson plans for eight key topics illustrate the development of problem-solving strategies and attitudes through the use of concrete and semi-concrete materials. Four lesson plans are then provided in which the content area is subordinate to the major focus on problem solving. Chapter 4 highlights strategies to encourage the use of instructional materials. Appendices include responses to a questionnaire concerning the mathematics curriculum, background information on manipulative materials, and a summary of their expected role. (MNS)

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INSTRUCTIONAL MATERIALS FOR ELEMENTARY MATHEMATICS

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UNESCO-NIER REGIONAL PROGRAMME

INSTRUCTIONAL MATERIALS FOR ELEMENTARY MATHEMATICS

Final Report of the Regional Training Workshop for the Development of Materials for Mathematics Education in Asia and the Pacific

Section for Educational Cooperation in Asia
National Institute for Educational Research (NIER)
Tokyo, Japan
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Summary of the Report

This report of the workshop fairly represents the issues and problems in promoting the use of environmental and low cost materials so as to motivate a problem solving approach. The deliberations of the workshop have revealed the variety, complexity and immensity of the problems to be overcome, and as indicated by the range of strategies advocated in Chapter IV, account must be taken of the common and differing characteristics of countries within the region. For example, prevailing socio-economic conditions have a definite bearing on the need to use low cost materials.

It was a commitment of the workshop to identify such materials which are readily available at low cost, in sufficient quantities, to be used by children in classrooms in any country within the region. Many of these fulfil similar functions to that provided by commercial materials in some countries.

Materials which reflect the impact of new technologies are often high cost items whose use is not feasible in some countries now. However, as costs reduce and as priorities within countries change, their future use can reasonably be anticipated and so there is passing reference to such materials in this report. Participants were conscious that links were to be made between each item of environmental and low cost materials as it was identified, and its potential to supprt a problem solving approach to teaching.

In essence, the workshop deliberations concentrated on the following two broad areas, embodied in the four objectives listed in Chapter I :

1) Identification of environmental instructional materials and their illustrative use in teaching mathematics, and strategies for increasing acceptability and use of these materials;
2) Use of instructional materials in problem solving.

The interdependence of the two areas is obvious, and the challenge to achieve a synthesis was accepted. Two distinct ways which illustrate the forging of these links are found in Chapter III. Key topics 1 to 8 generally provide exemplary treatment of universally taught areas of elementary mathematics in a format where attention is given to the development of problem solving strategies and attitudes through the use of concrete and semi-concrete materials. Subsequently, four exemplars are provided in which the content area, though recognizable, is subordinate to the major focus on problem solving. In

both cases the treatment is indicative and obviously not inclusive.

The participants did not reach consensus on all aspects of what constitutes problem solving, particularly in the context of elementary mathematics education. The working definitions included in Chapter II are deliberately inprecise. Although there was agreement on the general description of a problem solving process, some differences emerged when finer detailing was attempted. There was general agreement on the need for further work in this area, and the recommendation for further regional cooperation and development in Chapter IV was one outcome.

The image of the teacher as a problem poser, tolerant and supportive of divergent views and able to salvage useful outcomes from errors and false trails is to be nurtured. By encouraging children to think for themselves and to develop their own solutions, and then discussing with them their partial, unsuccessful or successful solutions, the teacher is more likely to develop problem solving abilities.

In particular, the participants wish to dispel any belief that problem solving is some kind of abstract, intellectual process in which concrete materials are of minimal value. The exemplary lessons should provide a range of counter examples.

In identifying a range of problems inhibiting the use of materials and some strategies which may be effective in overcoming them, which formed the basis of Chapter IV, the participants are aware of the variation which exists within and between countries of the region. The list was made as extensive as possible so that it could then be abridged for application within any part of the region.

Participants of the workshop are aware that the use of instructional materials in the teaching of mathematics is progressing well in many countries. It is important that future actions acknowledge the gains made so far and build upon them.

It is emphasized that, whatever mix of strategies is found appropriate for local application, it is hoped that they will be inclusive of both the wider use of instructional materials and the development of flexible, problem solving children who will be adults of the twenty-first century.

Chapter I Introduction

Background

The National Institute for Educational Research (NIER) of Japan organized jointly with Unesco in 1983 the Regional Design Workshop on Mathematics Education. Through a review of the state of the art of mathematics education in the region, the participants of the workshop identified several crucial issues and problems in mathematics education and suggested that regional cooperative actions in some areas be undertaken. The use of learning resources was one of the areas identified for further regional cooperative study. The workshop revealed that textbooks constitute the main resource for teaching and learning and suggested that steps be taken to encourage teachers to use more widely other learning resources. To this end it was urged that teachers should be made aware of how to prepare and use these resources in the classroom.

In view of the importance attached to learning resources in relation to the improvement of elementary mathematics education, NIER decided to organize, jointly with the Asian Centre of Educational Innovation for Development (ACEID), Unesco, Bangkok within the framework of the Asian Programme of Educational Innovation for Development (APEID), a Regional Training Workshop for the Development of Materials for Mathematics Education in Asia and the Pacific from 30 October to 17 November 1984.

Objectives of the Workshop

The objectives of the workshop were :

1) To identify key mathematics topics at the elementary school level which need the development and use of appropriate and concrete resource materials including low cost and environmental materials;
2) To examine and suggest effective ways and methods of using such concrete resource materials in the teaching/learning process of certain key topics;
3) To consider a general framework and work out strategies and

guidelines for the development of instructional aids and materials, particularly for developing problem solving skills in mathematics education at the elementary school level ; and

4) To cooperativelely develop some exemplars of such aids and materials to illustrate the suggested strategies and guidelines.

Participation

The workshop was attended by twenty participants from eighteen countries. Two resource persons, one representative from Unesco and two observers also attended the workshop. A list of all participants and the NIER Secretariat is included in Appendix I.

Inauguration

The workshop was opened by H. Kida, Director General of NIER. He extended a warm welcome to all the participants. He thanked Unesco and ACEID for their collaboration in organizing the workshop, and also thanked the Japanese National Commission for Unesco and the Ministry of Education, Science and Culture of Japan for their continuing support for the regional programme conducted by NIER.

N. Nishizaki, Deputy Director General for International Affairs, Minister's Secretariat, Ministry of Education, Science and Culture of Japan, welcomed the participants on behalf of the Ministry of Education, Science and Culture as well as the Japanese National Commission for Unesco.

E. Jacobsen, Programme Specialist, Division of Science, Technical and Vocational Education, Unesco, welcomed the participants on behalf of Unesco and thanked NIER and the Government of Japan for their continuous efforts and the cooperation extended to Unesco and to APEID in particular.

Officers of the Workshop

The workshop elected the following office bearers :

Chairperson :	Robert J. Gaffney (Australia)
Vice-Chairpersons :	Syed Ijaz Hussain Bokhari (Pakistan)
	Isabel Castro (Philippines)
Rapporteurs :	Joan Paske (New Zealand)
	Goh Mei Lian (Singapore)

Procedures of the Workshop

The first two plenary sessions after the election of office bearers were devoted to the presentation and discussion of the papers presented by the Unesco representative and one resource person which dealt with the practical as well as theoretical framework of the learning resource in elementary mathematics education. Papers prepared by participants dealing with the development of mathematics topics using concrete instructional materials were examined in depth in the group sessions to follow. Eight key topics were identified and individual/group assignments were given to develop exemplar materials for the teaching of these topics and problem solving. Substantial other work was also done in group sessions and lists of members of working groups are given in Appendix II. The workshop further discussed, in both group and plenary sessions, strategies which could be used to encourage classroom teachers to utilize instructional materials in their day-to-day teaching practice of mathematics.

Summary tables of the answers to the additional questionnaire, submitted by the participants prior to attending the workshop, were presented to the plenary group. This provides an information resource to supplement the report of the 1983 workshop, and is incorporated in this report as Appendix III.

During the workshop, the participants were given opportunities to learn about the state of the development and use of instructional aids and materials in Japan, as well as visiting several educational institutions concerned with mathematics education.

Final Report

The draft final report was presented and adopted at the closing session on the last day of the workshop with modifications to be incorporated in the final report.

Chapter II Priorities in Teaching and Learning

Introduction

Before coming to the workshop, participants were asked to select two or three key topics from among the mathematics topics taught in elementary schools for which it was considered necessary to use concrete instructional materials and aids in the teaching of them. In selecting the mathematics topics, participants were told to refer to the *Grade-wise Mathematics Topics* shown in Table 9 on pp. 18-25 of *Mathematics Education in Asia and the Pacific*, the final report of the workshop organized in 1983. They were asked to prepare a short paper indicating the instructional programme/lesson plan to be followed in the teaching of each of the selected topics, the instructional materials to be used in the development of the lesson, and the rationale for using these concrete instructional materials.

All participants brought with them some exemplars of aids and instructional materials listed in their papers, and many of them were incorporated in the exemplar lesson plans developed by the participants during this workshop. To examine the use of such concrete materials was one of the major objectives of the workshop.

Another of the major objectives was to explore ways in which the problem solving approach can be used when teaching mathematics at the elementary level. Before participants broke into their first working groups, they were addressed by E. Jacobsen and D.K. Sinha (see Appendices IV and V). Jacobsen reminded the participants of the two principal objectives when learning mathematics:

a) to recognize situations in which mathematics exists in society in order to develop practical problem solving skills;
b) to establish cognitive structures of mathematics.

"The workshop has thus two tasks," he said:

a) to develop teaching/learning materials which will assist in achieving the two above objectives; and
b) to think of real situations which may evoke the mathematics which we think important for children to learn, and which may invoke problem

solving skills.

He summarized what research has shown about the utilization of manipulative materials which involve pupils in the process of *doing* mathematics or at the very least, *seeing* some being done. However, Jacobsem warned participants about teaching the structural content aspects of mathematics by using only structured manipulative materials such as place value blocks and cuisenaire rods. "We must not forget those aspects of mathematics more directly related to the natural sphere of the learner. The personal experiences of the child should be more important than the external demands of the structure of mathematics." he said.

Sinha in his address said that the nature and complexion of instructional materials used for teaching mathematics in elementary schools has changed over the years. This, he said, can be attributed, in the main, to changes in emphasis in what is taught at this level.

The use of resources which are traditional in spirit and content is on the decline as is also the use of resources steeped in the so called "new maths". The spin off from this has been excellent in that it has triggered off rethinking about mathematics education in the elementary years. "Activity" which came in the wake of new maths is being reshaped to give a wider dimension. With the refining of the mathematics curriculum "activity learning" or "guided discovery learning" has gained new momentum.

There is ample research evidence to support the use of this approach. Activities which involve the children in manipulating materials, discussing their observations and forming conjectures, and which call on their everyday experiences, lead to the creation of a desirable climate in which children will learn mathematics. Of course, using this approach calls for the provision of the necessary materials.

The environment of the children can provide, for the resourceful teacher, not only many materials which can be adapted for use but also motivation for learning, since the children will learn better what they see to be relevant.

Sinha went on to say that there are three distinct features which affect the teaching-learning situation. First, the child's ability to communicate in words about mathematical ideas. Second, the ability to model the ideas and third, the gradually build-up ability to use modern computational aids such as calculators and computers.

Although it has not been required, as part of the report, to analyze or promote the use of calculators and computers as instructional materials, the exclusion of such an analysis should not interpreted as rejection of either. Children will need to be able to use both of them as problem solving tools in school and subsequently in adult life. Computers, in particular, have the potential, as yet largely unrealized, to fill a variety of roles in supporting learning in mathematics and as a significant factor in the direction of change

needed in the future mathematical curricula. There is a need for regional co-operation in sharing the design and results of research and practice in the application of computers to mathematics education.

In preparing instructional materials to use in active learning situations, teachers of today should set aside fears about the possible dilution of mathematics or about children developing wrong or distorted perceptions which will have to be unlearned at a later stage.

Manipulation of materials and a problem solving approach, taken together, hold the key to success in mathematics learning at this level.

Rationale for the Use of Instructional Materials in Teaching/Learning Mathematics

Mathematics is an abstract subject since the elements which it manipulates are themselves abstract. This makes mathematics difficult to teach as well as to learn. Concrete/semi-concrete materials are effective means to develop the concepts and principles. Their use will help children understand the underlying structures and principles of concepts and skills.

As children go through the process of manipulating and putting together concrete materials, mathematics becomes more meaningful and learning becomes more permanent.

Use of manipulative materials will provide children with opportunities to explore ideas and find ways of solving problems. In the process of manipulation, they may well find various kinds of mathematical relationships.

The sense of achievement and the elation of success will, in turn, promote interest in the learning of mathematics.

Instructional materials in the form of games will provide opportunities not only for practising and reinforcing skills learnt, but also for applying the concepts and principles.

Problem Solving Approach

First, what is a problem?

A problem is a challenge acccepted by an individual or group to whom no way of finding a solution is immediately obvious. It is the absence of *an obvious way* which distinguishes a problem from an exercise. For one person, the challenge may be an exercise, for another the same challenge is a problem. In some of the literature *exercise* is also described as a *routine problem*.

Many tasks stated in words are indeed problems because, for example, operations are not stated symbolically but in terms of the working definition used here, the two are not synonymous. Some tasks stated in words present

familiar situation and an algorithm for their solution is known to the solver. Such tasks are really exercises used to apply an acquired skill. It is possible and at times desirable to pose problems other than in the form of words. Pictures, concrete materials and symbols can all pose challeges both implicitly and explicitly.

What Then is a Problem Solving Approach?

The teacher is the poser of the problem, the facilitator who provides the concrete materials, asks questions, and who helps the children to share their experiences, successful or not, with their peers. There is a classroom climate where open communication, creativity and experiment can occur.

The teacher encourages and values persistence and divergent thinking and guides children in their rethinking if an impasse has been reached. Children will be given time to form individual or group conjectures, sharing these with others, and the teacher will accept that there will be variation in the time required for this to happen.

On accepting a problem to be solved the child will be immediately involved in certain mental processes. For some the problem as presented may require clarification. An active search is carried out through structure already known and attempts to match them with the unknown parts of the problem are made. A plan is devised which will require the use of some particular mathematical skills and strategies.

The plan is executed and the result evaluated. More than one cycle through the process may be required.

By mathematical skills it is meant such things as recall of number facts, knowledge of place value and knowledge of properties of some shapes. There are of course many others. By strategies it is meant such actions as making a model, drawing a picture, guessing and checking and making up a similar but simpler problem.

Children will not necessarily be aware that they are going through processes, and during their early years may need considerable guidance. The teacher should be deliberately building up the child's inventory of both skills and strategies. With continued use of the approach, they will develop systematic ways of tackling each new situations presented and become efficient problem solvers.

Key Topics

The papers from each participant were presented and discussed during the first group session. The topics presented covered the following areas:

Concept of Whole Numbers;

The Four Operations (addition, subtraction, multiplication and division) on Whole Numbers;
The Numeration System and Place Value with Whole Numbers;
Common Fraction Form of Fractional Numbers — the Concept and Representation of Fractions;
Addition, Subtraction, Multiplication and Division of Fractions;
Concept and Representation of the Decimal Fraction Form of Fractional Numbers;
Addition, Subtraction, Multiplication and Division of Decimal Fractions;
Ratio/Proportion;
Percentages;
Angles;
Area of Triangles;
Money;
Plane Geometric Figures;
Spatial Geometric Figures;
Simple Word Problems; and
The Process of Problem Solving.

Some of these topics were presented by more than one participant.

As it was not possible to study and include in the report all of these topics, there was a need to identify the key topics for the purpose of this workshop. After much discussion and deliberation, it was decided that the key topics for this workshop would be as follows:

1) The Concept of Whole Numbers including ordering them, writing numerals and patterning with them;
2) Addition-Subtraction and Multiplication-Division of Whole Numbers, with reference made to properties of addition and multiplication;
3) The Concept of Fractional Numbers including ordering them, writing them as fractions and finding equivalent fractions;
4) Addition and Subtraction of Fractions with a) Like Denominators, b) Unlike Denominators;
5) The Concept of Place Value to the Right of the Decimal Point along with operations on numbers with up to 3 decimal places;
6) Measurement of length, capacity, weight, time, money, angles, area;
7) Rounding, Approximating, Estimating;
8) Geometric Figures — a) Plane, b) Spatial.

It was felt that these topics encompassed the main ideas covered in the papers presented and that most mathematics taught in elementary schools could

be fitted under one or other of the headings.

Rounding, Approximating, Estimating was identified as one of the key topics though none of the participants had prepared a paper on the subject. It was, however, agreed that there is a need to develop instructional materials for the teaching of this topic.

Problem Solving was included as a separate topic since it was an area of major concern for this workshop. It was accepted that there is a need to improve mathematics teaching/learning through using a problem solving approach.

Exemplars of Instructional Programmes

In the second group working session, participants were asked to:

1) Examine and suggest effective ways and methods of using concrete resource materials in the teaching and learning of the key topics identified; and
2) Develop some exemplars of instructional programmes/lesson plans using a problem solving approach.

The exemplars prepared are presented in Chapter III of this report.

It was not possible in the time available to develop fully all of the key topics identified or to show the complete development of any one topic but sufficient were prepared in the agreed manner, *i.e.*, using concrete materials within a problem solving mode to indicate the direction in which mathematics teaching should go.

Problems and Issues in Using Resource Materials

The factors which inhibit change in attitude towards and practice in the use of instructional materials for teaching mathematics were identified in the third group session, and participants were asked to suggest ways of overcoming these factors.

The result of these deliberations forms Chapter IV.

Instructional Materials

Participants at the workshop were asked to pay particular attention to finding and using, in their exemplars, everyday objects with which children would be familliar.

It was recognized that there would be some differences country by country in the materials available, but most children and teachers would have access to at least some of the materials in the list presented.

It was also recognized that there are some mathematical topics which are best explored through the use of structured materials. Such materials, however,

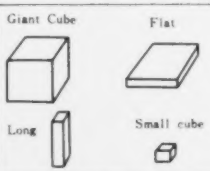
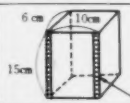
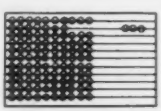
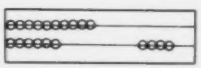
are not available in all countries represented at the workshop. The most useful and commonly mentioned structured equipment can, however, be replicated in paper or cardboard by the teacher or children and these materials are illustrated later in this chapter.

The following are the materials used in the exemplars shown in Chapter III.

A. Environmental Materials Used

1) Shells, pebbles
2) Fruits, leaves, seeds
3) Sticks, twigs, popsticks/ice-cream sticks, wooden strips
4) Beads
5) Paper, cardboards, newspaper clippings, wrappers of all sorts
6) Rubber bands
7) Strings, ropes, strips of cloth, wool, thread
8) Baskets, trays
9) Water, sand
10) Bottles (plastic)
11) Replicas can be made of rulers made from cards (100cm, 50cm, 30cm)
12) Money-Replicas of notes and coins appropriate to the country can be made from cards.

B. Commercially-Made Materials

Name of Material	Illustrations	Remarks
1. Place Value Blocks		Easily made out of squared paper. Refer to teacher-made materials No. 3
2. Rectangular Plastic Vessel		
3. Beaker		
4. Bead Frame		Can be easily made by teacher using beads and strings, or sticks. 

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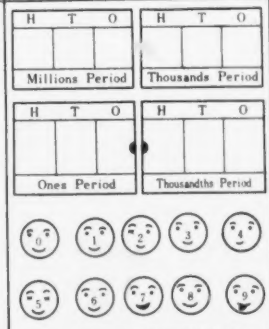
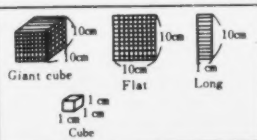
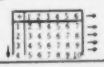
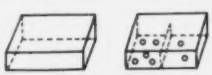
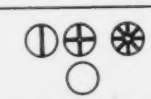
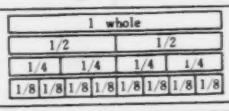
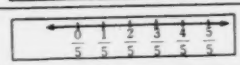
C. Teacher-Made Materials

Name of Material	Illustrations	Remarks
5. Abacus	The prongs represent 'place' and the counters 'face' values so children can model numbers.	Easily made from such things as clay, sticks and pieces of drinking straw, or the ribs of banana leaves can be used for the base.
6. Place Value Cards and Digit Faces	 4 of each digit face	Made on card or paper or such materials as vinyl floor covering. Each board is separate ; one board for each period. The boards can be of any size but 15 cm x 12cm is quite convenient. The digit faces should be able to fit into the columns marked on the boards.
7. Place Value Blocks		Can be made on squared paper and glued to card.
8. One Hundred Board/Chart		Can be made on paper. Without the numerals written in, it can be used for teaching decimals, percentage prime and composite numbers and factors or for showing addition or multiplication arrays. 
9. Subtraction Box		A small box such as a match box divided into two sections as shown. The partition in the middle should be made lower than the sides, so that beads from one section can move to the other when the box is shaken.
10. Fraction Discs (1/2, 1/4, 1/8) (1/3, 1/6, 1/12) (1/5, 1/10)		Made by using vanguard or stiff plastic sheets or other suitable materials. The reverse sides of the discs can have magnetic tape attached if available.
11. Fraction Strips (1/2, 1/4, 1/8) (1/3, 1/6, 1/12) (1/5, 1/10)		Made by using vanguard sheets or other suitable materials.
12. Fraction Line Cards (1/4, 1/8, 1/6, 1/5, 1/10, 1/12)		All the fraction materials can be made by children.

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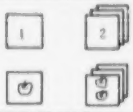
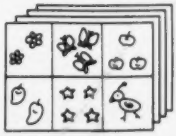
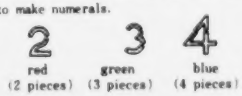
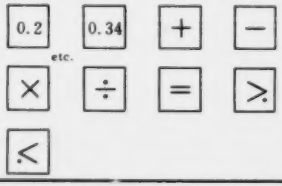
Name of Material	Illustrations	Remarks
13. Picture Cards for Matching : Numeral Cards, Picture Cards		The pictures can be of anything of interest to the children. Stylized pattern arrangements and random arrangements should be used.
14. Dominoes		Can be made as shown or can have pictures to be matched.
15. Bingo Cards	 A cube with numeral 1, 2, 3, 4, 1, 3 on each face.	The die is thrown and children covers the section showing the correct number of pictures.
16. Jig-Saws	a) Children arrange cut-out part of numerals to make numerals.  2 red (2 pieces) 3 green (3 pieces) 4 blue (4 pieces) b) Children match appropriate 2 pieces.	The Jig-saws can be more complex and show +, −, × or ÷ facts which are completed, when the pieces are fitted together.
17. Picture Cards	Front Picture Back Sand paper	Paste picture cut-outs on card and on the reverse side paste a piece of sand paper so that it can stick on child's clothing or on a flannel board.
18. Number, Operation and Relationship Signs on Cards	 0.2 0.34 + − etc. × ÷ = > <	Many cards showing numerals must be made. Only 2 examples are shown here. The small dot below > and < will help chidlren see which sign is meant.

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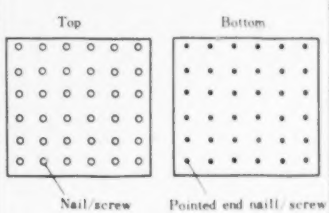
Name of Material	Illustrations	Remarks
19. Geoboard		A simple one can be made easily by using a piece of thin wood and piercing 36 nails/ screews through it at equal interval as illustrated in the diagram. Children can use the side of the board with the pointed nail/screw to mark points on paper. The points will help them to join and get the shapes required. In the same manner, children can use this teacher made geoboard to mark points on plywood and make their own geoboards.

Chapter III Exemplar Lesson Plans

Introduction

For each of the key topics identified, exemplar treatments were developed, in which commitment to a problem solving was taken into account. Included in each teaching/learning sequence are problem solving strategies and the use of concrete materials. The materials used in each of the teaching sequences are illustrated in the previous chapter.

It is recognized that the class/age level shown vary slightly from country to country as may the choice of environmental materials used. The word *set* is used in these learning sequences to denote a group of objects being manipulated in some way, not in the more precise mathematical way.

No reference is made to the use of textbooks as instructional aids since it was agreed that teachers are already familiar with their use and indeed, in most countries, rely on them as the only instructional material.

Exemplars for all the key topics identified were not written to the complete sequence within any topic, but it is hoped that those which follow will provide a guide to teachers wishing to use the approach.

Key Topic 1 Concept of Whole Numbers

Sub Topic 1 The Idea of Four and Its Numeral

Class/Age Level: Grade 1; 6 years

Objectives: Children will be able to:

1) Tell the numbers of objects using both concrete ones and picture in a collection of up to 4;
2) Match numeral cards of 4 with the collection of 4 objects, both concrete and picture.

Instructional Materials: Environmental objects such as leaves, seeds, etc., number cards, picture cards.

Learning Sequence:

Step 1: Give each child her/his own objects to make collections of one,

two and three.

"What might be the next collection? Why did you say four? Make sets of 4 with your leaves, seeds and sticks. Make each set into a different pattern. Look at the other patterns made. Who has most in a set? Who has least? How do you know that the sets have 4 in them?"

Step 2: "Find 3 pictures each with 4 things. Match your sets with the pictures. Are there any objects left over? Why not?"

Step 3: "Who knows how we write the word for 4 objects? Spell it all of you. The word has quite a few letters. Does anyone know we can write about 4 using a numeral?"

Show children how to write the numeral if no one offers.

On-going evaluation: "Make some sets of 4 objects. Make a different pattern with each and match each pattern with a numeral card. Draw the patterns you make."

Follow-up Activities:

1) Guessing game

A pair of children take turn holding a certain number of counters (not more than 4) and the other guesses the number of counters in the hand. If he is right he can have those counters. Winner has most counters after 4 turns each.

2) Play Dominoes (See page 15.).
3) Play Bingo (See page 15.).
4) Play Jig-saw (See page 15.).
   a) Children arrange cutout part of numerals to make numerals.
   b) Children match appropriate 2 pieces.
5) Play Pairing (See page 15.).

Children match picture cards with numeral cards, or *play Memory*. Put all cards face down. In turn children turn up 2 cards keeping the cards if they are a matching pair.

On-going evaluation:

1) Have children pick up objects and/or picture cards to match a given numerals (1, 2, 3, 4).
2) Have children pick up a number card to match a given picture card or collection of not more than 4 objects.

Sub Topic 2 Comparison of Two Numbers

Class/Age level: Grade 1; 6 years

Objectives: Children will be able to:

1) Match 2 sets of objects to see which has more, which less;
2) Talk about the matching;

3) Write number sentences about the matching using the symbols $>$ $<$.

Instructional Materials:

1) Counters or beads or leaves or any such materials.
2) 2 shallow baskets or dishes or broad leaves.
3) Short narrow strips about 4cm long. Each child needs several such strips.
4) Number tickets.
5) Sentence cards (See page 15.).

Learning Sequence 1:

Step 1: Put some of the objects on to each dish.
"How could we find out which dish has more?"
Let children experiment and show their idea to a neighbour.
Ask several children to explain what they did. (If child has 2 matching sets, adjust one of them.)

Step 2: Put your dish with more on the left and put the one with less on the right.
"Which has more? How do you know?"
"Tell your neighbour about your 2 sets in this way. I have more in the left set because 7 is more than 5, etc."

Step 3: Ask children to write or write yourself: Seven is more than five.
"What a lot of letters and words! Who can find number tickets for the numbers?"
"Put the tickets near the dishes. Does anyone know a sign we could make which means *is more than*? Let us make that sign with 2 of the little strips. The 7 is biggest, so we put the big space against it. It's like a crocodile opening its mouth wide to swallow the big number."
"Make your strips like a crocodile mouth. Put the sign between the seven and the five. Read what it says."

On-going evaluation: Make up some different number sentences about *is more than*. Show each sentence with objects in the dishes and make the sentence with digit and strips. Remember to check that there really are more in the first set.

Learning Sequence 2:

Step 1: "Show me $8>6$ with your objects, tickets and strips. Find a card which has $>$ on it."
Note: The dot below the $>$ will help children decide which sign is meant.
"Put your tickets on the card to finish the sentence. Now practise with your partner. One makes a sentence on the card.

The other shows it with objects, tickets and strips."

Step 2: Repeat the sequence this time putting the smaller set on the left. Accept such words as *smaller than, fewer than, not as many as* or even *littler than* from the children at this stage.

Extension Work for Very Able Children:

Step 1: "How coud we rename 7? Show me the pattern for that name. Are there more patterns? Write about your patterns like this or use tickets."

e.g. [5] and [2]

[3] and [4]

Step 2: "Now let us rename 5 in the same way. Show each pattern and use tickets or write about it like you did for 7."

e.g. [3] and [2]

[1] and [4]

"Who thinks they could use these names for 7 and for 5 to write *more than* sentences? Remember to check that you are speaking the truth."

Sub Topic 3 Ordinal Numbers

Class/Age Level: Grades 1 and 2; 6 and 7 years

Objectives: Children will be able to:

1) Name the position of an object starting from the left or the right with up to 7 objects;
2) Name the position before and after any given position;
3) State how many are in front of any given position;
4) Explain how the starting point is important.

Instructional Materials:

1) Children themselves.
2) Everyday objects such as fruits, stones, seeds, sticks, shells, etc.
3) Worksheet showing a slide with 7 children in various positions.

Learning Sequence 1:

Step 1: Ask children to make a row of like objects.

"Choose one of your objects. How will you explain to somebody which object you have chosen?"

Let children explain their own ideas.

"Is it possible to explain which object by using a number? Will a counting number do? Why not? Where do we say someone who wins a race comes? How do we talk about the person who

finished next to first? Next to second? How do we know a person is first?"

Step 2: Choose 5 children to come out to the front.
"Arrange yourselves in a line so that there is a first, second and third. What about the other two? What will we call them? Which way is the line facing? If they all faced the other way would *x* still be first? Where would she/he be?"

Step 3: Choose a particular object.
"Make a row of 6 objects so that your special one is 3rd from the left. Now move it so that it is 3rd from the right. Move it so that it is 5th from the left. Now move it so that it is 5th from the right."

Step 4: Choose a different set of 7 children. Name one of the girls.
"Put yourself in a row facing the class. Now turn to face me. (Teacher stands at the left end of the row.) Who can put (girl) so that she is 4th in the row? Now turn about. Who can move (girl) so that she is still 4th in the row? How far did you move her? Why didn't you have to move her?"

On-going evaluation: Draw a set of 7 objects. Put a ring around the 4th from the left and from the right.

Learning Sequence 2:

Step 1: Choose a different set of 6 children.
"Put yourself in line, and turn one way. Which way have you turned?"
Choose one of the boys.
"Who can move (boy) so that he is 3rd in the line? Who is next to him? Is that the only person? There are 2 people next to him but who is in front of him? Who behind him? If it was a race where was the person in front of (boy) coming? Where the person behind?"

Step 2: Make a set of 7 objects. Counting from the left move the 5th up a bit.
"How many objects are in front of the 5th? How do you know? Repeat with other positions."
Then repeat Step 2 with counting from right.

Step 3: Have children work in pairs. In turn children make a row of 7, select one object and describe its position to partner using position and direction, *e.g.* "My object is 3rd from the right, what it is?"
If partner correctly identifies object they exchange places.

On-going evaluation: Give out worksheet showing slide with children. Give instructions orally such as:

1) Put a box round the first person from the left; 2) Colour the third person blue; 3) Who is 5th from the front? 4) Who is 5th from the back? 5) Who is in the same position from front and back? 6) How many people are in front or the 6th person? the 7th? 7) What position is Noriko? Mei Lin? Sara?, etc.

Sub Topic 4 Place Value in the Ones and Tens Places

Class/Age Level: Grade 2; 7 years

Objectives: Children will be able to:

1) Name the single digit numbers 0, 1, 2, ····· 9;
2) Know that after 9 you have to move out of this ones place;
3) Know that the next place to the left is called *Tens*;
4) Model a number with 2 digits;
5) Explain which is the tens place, which the ones;
6) Show that when a digit moves into the tens place from the ones it becomes 10 times bigger;
7) Write about a 2 digit number showing tens + ones;
8) Write about a 2 digit number using the shortest form.

Instructional Materials:

1) 11 sticks or ice cream sticks or long thin leaves for each child.
2) 11 rubber bands for each child.
3) Place value blocks or replicas (See page 13.).
4) Place value cards and digit faces (See page 14.).
5) Tens-ones boards for each child.

Learning Sequence 1:

Step 1: Give out sticks, rubber bands and tens-ones board.
"Put sticks into the spaces in the ones column. How many fit? What might we do if we have one more stick? Put a rubber band round a bundle of ten sticks and place the bundle in the tens column. What does it show now?"

Step 2: Get children to continue to place sticks in the ones column making a bundle every time they have 10 and moving it into the tens column until all 9 spaces are filled with bundles.
"What number is shown now? If we fill all the spaces in the tens and in the ones column, what number will be shown? How many digits?"

Step 3: Give practice in showing other numbers.

e.g. 1) "Show me 83. How many bundles of 10? How many ones? Write it for me 8 tens and 3 ones."
2) "Show me 60. How many bundles of 10? How many ones? Write it for me 6 tens and 0 ones."
3) "Show me 7. How many bundles of 10? How many

ones? Do we put zero in front of the 7 ?"

Step 4: Give out place value blocks or replicas.
"We will say today that the long stands for 1. Show me 7. What shape will we use for 10? Show me 77. How many longs fit on a flat? How many 7's in 70."

Step 5: Give practice in showing other numbers.
e.g. "Show me 69. How many flats? How many longs? Write it 6 tens and 9 ones. Write it in short from 69. Read it both ways."

Step 6: Use ones board and sticks: No rubber bands, and place value cards and faces.
"When we used place value blocks we only used one flat for 10. Do you think we could use just one stick to show 10? Where would we put it?"
"Show me 34 with sticks on the board. Find digit faces for 3 and for 4. Where could we put the 3 face so that it stood for 30? What is its face value? What is its place value? What is its total value? Where could we put the 4? Why is it not worth more than the 3, its face value is more?"

Step 7: Put children into group of 4 if possible. One tells a two digit number; one shows with blocks; one shows with sticks on tens-ones boards; one shows on place value cards.

Key Topic 2 *Addition-Subtraction, Multiplication-Division of Whole Numbers*

Sub Topic 1 **Introduction to Addition**

Class/Age Level: Grade 1; 6 years

Objectives: Children will be able to:

1) Put together 2 sets of objects and talk about what they did;
2) Show adding to sums of five by putting together 2 sets;
3) Draw a picture about the putting together.

Instructional Materials:

1) Everyday objects such as fruit, vegetables, small toys, nuts, stones, etc.
2) Coloured cutout pictures of the everyday objects. Pictures backed with piece of sand paper.
   Note: At least 10 of any kind will be needed.
3) Flash cards showing(i)and(ii)→(iii) + (iv) =.
4) Flannel board.
5) String or wool or thread for putting round a set of objects.
6) Chart with pocket for holding objects and/or pictures.

Learning Sequence 1:

Put objects or pictures into pocket of chart before the lesson begins.

Step 1: "Find me one green mango (or some other chosen object). Put it on the table. What do we start with?"
Get children to describe the object as well as state how many.
"Find me one yellow mango (or some other chosen object). Put it on the table away from the green mango. What have we got now? Who can come and put these two sets together and tell us about what happened?"
Look for something like: "We had 1 green mango and I put 1 yellow mango with it. So now we have 2 mangos".
Repeat using different sets of objects. If possible children work in groups, choosing objects, describing them, putting them together and talking about what they did.

Step 2: "If we choose more than one object at a time to make a set, can we put the two sets together? Show me. If you start with a set of 3 white shells and put a set of 2 brown shells with it what do you have?"
Repeat using different sets of objects. Children working in groups telling the others about their actions.

Step 3: Use pictures on flannel board or draw pictures on black board. "What set shall we start with? Choose or draw it. What shall we put with it? How many will we have in the new set? How do you know?"
Show children how to show the sets put together.

Step 4: "Show some more drawn sets put together. What are the 2 sets ? What has happened to them ? Describe the new set."

On-going evaluation: *Either*, give children worksheet to complete, *or*, ask children to draw some pictures showing 2 sets being put together.

Learning Sequence 2 with the Following Objectives:

Children will be able to:

1) Interpret a verbal story and show it with materials and pictures;
2) Know that instead of describing the two sets and putting them together, they can write a number sentence about what they did;
3) Recognize the sign + as a symbol for *add* and the sign = as a symbol for *makes* or *is the same as* or *equals*;
4) Write and read addition sentences, modelling them with materials.

Step 1: If Taro has one yellow rose and Hanako has three red roses how many roses have they in all (between them, altogether)? Try to use different expressions so that children become accustomed to all types.
"What can we do to show that story? Show me. Read what you have shown."
"It takes a long time to read the story that way, doesn't it?"
"One yellow rose and two red roses make three roses altogether. What numbers were used in the story? Who can write them in the order they are used? Does anyone know what we could put between the numbers so that they tell a story?"
Responses may vary. Encourage diversity, *e. g.*, 1 and 2 makes 3; 1 joined with 2 matches 3; 1 and 2 is 3; etc.

Step 2: "Does anyone know the special sign we can use with numbers to show we have put the two sets together?"
Tell them if no one offers.
"Does anyone know the special sign we use with numbers to show makes, matches, is, equals?
Tell them if no one offers.

Step 3: Repeat steps 1 and 2 using different names and numbers. Make sure that each problem refers to a girl and a boy, or that equal numbers of problems refer to girls as well as boys.

Step 4: "You have been writing number sentences. Draw the shape a number sentence makes. Sometimes we want to show adding in a different way. Has anyone seen it written so that it makes a different shape?"
Show the *working form/column form* if no one offers.
Write some + number sentences on the board.
"See if you can write these in the new way."

On-going evaluation: "Each think of an interesting story where you have to add the numbers. Draw the pictures for your story. If you can write your story in words, then show your story using a number sentence. Lastly show your story in working/column form."

Sub Topic 2 Subtraction

Class/Age Level: Grade 1 or 2; 6 or 7 years
Objectives: Children will know:

1) That taking away is the opposite of adding;
2) That taking away can be shown by separating objects from a set to find how many left;
3) That taking away can also be shown by comparing to find how

many more or how many less.

They will be able to:

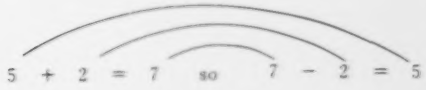
4) Use the sign for take-away;
5) Find everyday situations where subtraction is used;
6) Make up everyday word problems that use subtraction;
7) Write subtraction facts;
8) Write the related addition fact.

Instructional Materials:

1) Everyday objects such as shells, leaves, little toys, etc.
2) Bead frame (See page 13.).
3) Charts showing separating and comparing.
4) Flash cards.
5) Subtraction box (See page 14.).

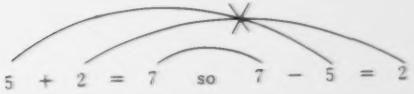
Learning Sequence 1:

Step 1: "Make a set of 5 objects. What are you starting with? Now make another set, this time with 2 objects. Put the set of 2 with the set of 5. Tell me the addition sentence. Did you finish with more or less than you started with? How could you get back to the 5 you started with? Show me. Tell me both sentences. Who knows how we can write about this taking apart? Tell me the take-away sentence you showed. Now tell me both the adding and the take-away sentences. Show a rainbow over the top to make sure you have told the truth."



Step 2: Give children more practice of putting together, then taking apart and writing about what they did using a rainbow to check.

Note: If children take away what they started with, the rainbow will be destroyed.



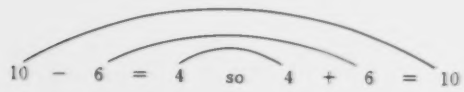
Step 3: Get children to show the putting together and taking apart using the bead frame.

Learning Sequence 2:

Step 1: Use everyday objects or bead frame to repeat Step 2 from previous lesson.

Show charts of pictures and/or diagrams depending on country.

"What do the pictures show? Who can tell the take-away sentence? Who can tell the adding sentence that belongs with it? Write both of them for me. Use the rainbow again."



Step 2, 3…as desired:

A: 1) Show children flash cards. Let them use everyday objects to find the difference. Check by turning over.

2) When more confident, children work in pairs. One shows card, the other supposes difference. If correct a card is given, if not the questioner keeps it.

B: 1) Show children subtraction box. Put beads in.

"How many went in?"

Shake a box to scatter beads into the 2 compartments. Open one side of the box.

"How many went in? How many showing? How many are hidden?"

Let them use objects to find out. Check by opening the box.

2) When more confident, children work in pairs. One puts beads in and writes on top of the box and other player opens one side and records number of beads showing. Then first player states number of missing beads and completes the sentence. If correct change places.

Learning Sequence 3:

Step 1: "Sam had 9 marbles. Heni won 5 of tem. How many left for Sam? Show me that story with shells. What did you start with, what did you do? Tell me the take-away sentence."

Repeat with other similar stories, then have children make up their own stories.

Step 2: Sin-Leng has 8 flowers. Tek-Hong has only 5 flowers. How many more flowers has Sin-Leng than Tek-Hong? How could we show this story? When we match 5 with 8, how many objects unmatched?" Show matching chart, *e.g.*, two flowers are not matched.

"Could we write a number sentence to find out how many not matched? What sign might we use?

On-going evaluation: Give more examples of stories asking "How many more?" or "How many less?" Get children to show or draw the two sets being matched and write the subtraction sentence. Get children to make up stories of their own about "*more*" and about "*less*".

Sub Topic 3 Subtraction with Renaming (Regrouping)

Class/Age Level: Grade 2 or 3; 7 or 8 years

Objectives: Children will be able to:

1) Rename a number like 63 as 50 and 13 or 50 and 10+3;
2) Show this renaming with materials;
3) Subtract a 2 digit number from another 2 digit number where renaming (regrouping) is required;
4) Make up stories in words about situation where this kind of subtraction would be needed;
5) Write the related addition equation for a completed subtraction of this kind.

Instructional Materials:

1) Place value blocks or replicas (See page 13.).
2) Tens and ones boards.
3) Sticks and rubber bands.

Learning Sequence:

Step 1: "We have a problem. We know how to take a number like 13 from a number like 48, but what if the story went like this! ——I saved 43 stamps and my friend wanted to buy 18 of them. If I sold the 18 how many would I have left? What would we do?"

Give the children the place value blocks and/or sticks, rubber bands and tens-ones board.

"Show me 43 with your material."

If using sticks they must be in bundles of 10 as well as some single ones.

"Find a way of taking 18 from the 43."

Step 2: Children share their solutions explaining what they did.

Step 3: "How would we write about what you did? 43−18=25 is a sentence. Let us write it in working form."

	T	O
	4	3
−	1	8

"What did you do so that you could subtract 8? If we strike out 4 and replace it with 3 in the tens column what must we put in the ones column?"

	T	O
	3	13
	~~4~~	~~3~~
−	1	8

"How have we renamed 43 to make it possible to subtract 18?"

Step 4: Give more practice in solving similar exercises. Children continue to use the materials until they are confident.

Note: These steps 1 to 4 would be repeated for several lessons.

Step 5: "What do we know about adding and taking away? If we can write $6+7=13$ so $13-7=6$, can we also write an addition sentence for subtraction with bigger numbers? Write the addition sentence that goes with $43-18=25$. Use the rainbow to check."

$43-18=25$ so $25+18=43$

"Show it with your blocks or sticks."

On-going evaluation: "Now use the subtraction box. Make up part of a word story: put beads in the box, shake, open one side, continue story. Now finish the story using working form for the subtraction, *e.g.*, I collected 32 leaves. Some of them were green, some red. 17 of the leaves were green, how many red? 15 leaves were red.

$32 - 17 = 15$ so $15 + 17 = 32$

	T	O
	2	12
	~~3~~	~~2~~
−	1	7
	1	5

Sub Topic 4 Multiplication of Single Digit Numbers

Class/Age Level: Grade 2 or 3; 7 or 8 years

Objectives: Children will know and be able to show:

1) That addition can be turned into multiplication if the addends are the same;
2) That multiplication of whole numbers can be turned into

addition;

3) That order is not important in multiplication.

Instructional Materials:

1) Everyday objects such as leaves, shells, stones, etc.
2) Flannel board and pictures.

Learning Sequence:

Step 1: "Show me this number sentence with your objects, $2+5=7$. What did you start with ? What did you do ? What did you finish with ? Tell me a story in words about $2+5$."

"Now show me $5+5=10$. How many in each set ? Now show me $5+5+5=\square$. How many in each set ? Try $5+5+5+5=\square$. Are there still 5 objects in each set ? Does anyone know or a quicker way we could write about 4 sets of 5 ?"

Note: In some countries children might suggest $5\times4=20$ meaning 5 taken 4 times. In other countries they might suggest $4\times5=20$ meaning 4 sets of 5.

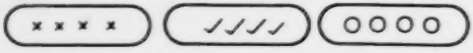
Step 2: Show 4×5 (5×4) on the flannel board using pictures. Show $4+5$ on the flannel board using pictures.

"Can we turn $4+5$ into times (multiply) ? Why not ?"

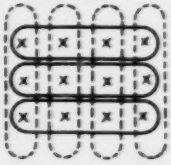
Step 3: Ask children to build up a table of a given number, *e.g.*, $\times6$ using their materials, writing the addition sentence then the multiplication sentence.

Step 4: "I have 3 kinds of sweets and I have 4 of each kind. My friend has 4 kinds of sweets but only 3 of each kind. Who has most sweets ? Use your objects to find out. Can you arrange your objects in a pattern so that you can show my sweets and my friend's sweets using only 12 objects ?"

Allow children to experiment. If they persist in putting them like the following, you might have to help with clues.



Help them to see that it is the number of objects that is important but not the objects themselves so they could be shown:



"If I look straight at the pattern what do I see ? If I turn my head sideways what do I see ?"

On-going evaluation: "Can we write about this special pattern using a number sentence ? What do you see looking straight? 3×4. What do you see with head turning? 4×3. Is there the same total each time ? Is it true then that $3 \times 4 = 4 \times 3$? Try it with some other numbers."

Sub Topic 5 Division of a 2 Digit Number by a Single Digit Number

Note: It is expected that children already know what division is, and know their multiplication division facts to 9×9, $81 \div 9$ and have used remainders before within the basic facts, *e.g.*, $36 \div 5$, $43 \div 7$, etc.

Class/Age Level: Grade 4 or 5; 9 or 10 years

Objectives: Children will be able to show with materials and using working form:

1) How to divide a number less than 100 by a number less than 10 with no remainder;
2) How to divide a number less than 100 by a number less than 10 with a remainder.

Instructional Materials:

1) Everyday objects such as leaves, stones, shells, etc.
2) Sticks in bundles of 10 and some single sticks.
3) Place value blocks or replicas (See page 13.).
4) Tens and ones cards.
5) Pencil and paper.

Learning Sequence:

Step 1: "Grand mama has 28 sweets to give to her 2 grand children. If they are to have equal shares, how many will each get?"
Ask children to solve the problem using their objects.
"How did you find out ?"
Give out place value blocks.
"Let the long stand for one. Show me what you did using the blocks."

Step 2: "That was quite easy. We can write it as a sentence or in working form."

$28 \div 2 = 14$

	T	O
	1	4
2	2	8

"How many 2's in 20? Will it do if we put 1 in the 10's column ? How many 2's in 8 ?"

Step 3: "Suppose grand mama brought 84 sweets to share between 4 grand children. Try it with your objects. Now use place value blocks. Which is quicker ? Do you get the same result ? Write it in working form. Remember to label your columns T. and O."

Step 4: "Why is it quite easy to solve $28 \div 2$ and $84 \div 4$? If grand mama brought 64 sweets to share between 4 grand children, what might we do ?"

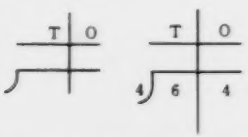
Give children bundles of sticks, single sticks and tens ones cards.

"Show me 64 with the sticks and cards. How many bundles ? How many single ones ? Where have you put the bundles ? If you have 6 bundles how many bundles will each child receive ? What might you do with the 2 bundles left ? Show me. How many single sticks have you now ? Do you know how many 4's there are in 24 ?"

On-going evaluation: Repeat Step 4 using place value blocks. Make sure that children actually exchange the 2 flats. For 20 longs and so not retain the flats.

"Does it come to the same when you use this material ? Now make me up another story like that and show it with materials."

Step 5: If we did not have materials and paper and pencil or board and chalk, how might we solve the problem. Label the columns. What are we dividing by ? What are we dividing into ? Put the numbers in the right places."

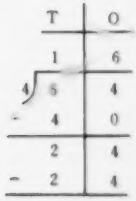


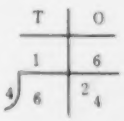
"How many bundles of 10 ? Where will we put the 1 ? How could we show that we took those 40 out and it left us with 20 ?"

"2 tens and 4 ones is the same as ☐ ones ? Finish the division."

Repeat Step 5 several times using other numbers where there is no remainder. Children should continue to prove their solution is correct by showing the problem with materials. This would extend over several lessons.

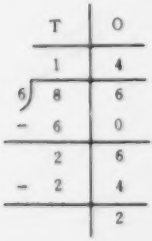
Note: In some countries the children will be expected to show as on the left, in others as on the right.

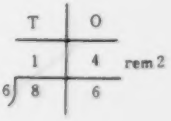




Later Development for the Objective 2:

Step 1: "So far all our divisions have worked out exactly. The grand children were able to share the sweets equally. Suppose grand mama brought 86 and had 6 grand children. How many would each child get ? Show me 86 with the blocks or with the sticks. How many lots of ten sweets can she give ? How many will she still have left ? Show me the working form so far."





"Will 6 divide exactly into 26 ? What is the nearest ? When you take 24 from 26 what will be left ? What might grand mama do with the 2 sweets left over ?"

On-going evaluation: Repeat Step 1 as often as necessary using different numbers.

Key Topic 3 Common Fraction Form of Fractional Numbers

Sub Topic 1 What is a Fraction: Using Denominations of 2, 4

Class/Age Level: Grades 2 and 3; 7 and 8 years

Objectives: Children will:

1) Find the approximate $\frac{1}{2}$ or $\frac{1}{4}$ of an object such as an apple;
2) Know that the size of a fractional part is dependent upon the size of the whole;

3) Fold their cup paper into matching pieces;
4) Know that they use a fraction to name the matching pieces;
5) Know that the fraction $\frac{1}{2}$ indicates that one of two matching pieces is referred to;
6) Know that the digit above the line denotes the number of matching pieces referred to and the number below the line denotes the number of matching pieces that the whole has been divided into;
7) Show the fractions $\frac{1}{2}$, $\frac{1}{4}$,$\cdots$ by folding any cutting paper;
8) Find $\frac{1}{2}$ or a square or rectangle or circle in more than one way.

Instructional Materials:

1) Paper for folding 6 oblongs, 6 squares, 6 circular shapes drawn on a worksheet.
2) Scissors.
3) Coloured pencils, crayons or felt pens.
4) Solid figures such as apples; knife to cut them.

Learning Sequence:

Step 1: Show apple or some firm fruit.
"I have only one apple and I want to share it equally with my friend. How many pieces will I need to cut it into ? Where shall I cut it ? How will I know if I have got a fair share ? Does anyone know what we call each piece ?"

Step 2: Cut an apple into 2 pieces taking care to see that the 2 pieces are not the same size.
"How many pieces have I got ? Have I got 2 halves ? How can you tell ?"

Step 3: "If I want to share this new apple equally, who wants to come and cut it so I do have a halves ?
Let children decide where the cut is to be.
"Do the pieces look as though they match ?"

Step 4: Put these pieces of apple aside.
"Each piece is about one half of the apple. Now let us use this bigger apple. Who thinks they can cut it in half ? Can you make the cut a different way through the apple ? Try it."
"Look at the size of this half of an apple. Does it match the other half of the same apple ? Now look at half of the first apple. Does it match the half of this second apple ? Why do you think the second half apple is bigger than the first half apple ? If I wanted half an apple that was smaller than this first piece, what would I have to start with."

Step 5: "Now let us take new apple and see if we can share it equally between me and 3 friends. How many matching pieces will we

need this time ? Where will we cut it so we get 4 mathing pieces ? Could we cut it any other way ? What shall we call each piece ?"

Step 6: Give each child a sheet on which is drawn 6 oblongs (same size), 6 squares (same size), and 6 circles (same size). Make sure that children have scissors and coloured pencils, too, but no rulers nor pencils.

Ask children to cut out all the shapes.

"Your task is to see how many ways you can fold each shape to make (i) halves, (ii) quarters/fourths. You may fold and then cut. You may not rule lines! Colour all the halves you make blue. Colour all the fourths/quarters you make red."

Give children time to experiment. Give hints if they are stuck but avoid directing.

Step 7: Make a chart showing all the shapes for half and all those for fourths/quarters.

Step 8: Have ready a worksheet of your own but much bigger. In front of children cut out some of your shapes. Fold and cut to show half and quarter/fourth.

"Have I cut half ? quarter ? How can you tell ? Why are my pieces bigger than yours ?"

Step 9: "We know how to write about whole numbers. Writing about two, it might look like this 2, about four like this 4. Does anyone know how we can write about a half ? a quarter ?"

"What does the 1 above the line tell us ? Could we have a 2 above the line ? Show me with quarters ? How many quarters are you showing ? What does the 2 below the line mean ? What the 4 ? Which are bigger $\frac{1}{2}$'s or $\frac{1}{4}$'s ? Is that always true ? What about my $\frac{1}{4}$ and your $\frac{1}{2}$? Now try to tell me again about $\frac{1}{2}$ being bigger than $\frac{1}{4}$. Yes, they have to be $\frac{1}{2}$ and $\frac{1}{4}$ of the same thing."

Further Development with the Following Objectives:

Children will be able to:

1) Show a set of 2 objects and identify $\frac{1}{2}$ and $\frac{2}{2}$ of the set;
2) Show a set of 4 objects and identify $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{4}{4}$ of the set;
3) Know that the size of the objects doesn't matter this time; that they don't have to match in size to be $\frac{1}{2}$ or $\frac{1}{4}$; but the subgroups formed do have to match in number.

Instructional Materials:

1) Everyday objects of varying size. Try to make them of interest to the children.
2) Sets of pictures about 6 to 8 cm long and 3 to 5 cm wide. Each

picture should be of a separate object and have a piece of sand paper glued to the back (See page 15.).

3) Paper, pencil, scissors, coloured pencils, crayons, etc.

Learning Sequence:

Step 1: "When we found $\frac{1}{2}$ or $\frac{1}{4}$ of an oblong or square, what was special about the size of the pieces ? Choose a set of 4 things. Why did you choose those 4 ? Tell your neighbour about your 4. Are they all the smale size ?"

"Now choose one of them. Don't let your neighbour know which you are thinking about. Describe your object to your friend. Can she/he identify it ? How much of your set have you been talking about ? The whole number 1 would tell you how many but I asked you tell me how much ?"

Step 2: Choose a child with clothing to which will stick the sand paper. If necessary have hanging up a flannel board. Ask a child to choose 4 of the pictures.

"How many have you chosen ? Why did you choose those ? " Stick them on the other child's chest or on the flannel.

"Are they all the same size ? How much of the set is on the chest ? Can we talk about it in quarters/fourths as well as saying all the set ?"

As you talk now lift pictures off and replace them.

4 out of the 4 are on the chest.

1 out of the 4 is (describe it, *e. g.*, an orange).

2 out of the 4 are (describe them).

3 out of the 4 are (describe them).

On-going evaluation: Choose your own set of objects or pictures. Show your neighbour $\frac{1}{2}$, $\frac{2}{2}$. What matches the number or the size ? How many in the whole set ? Can you show $\frac{1}{2}$ if you have 3 in the set ? What about 4 ?"

Sub Topic 2 Equivalent Fractions

Class/Age Level: Grade 4 ; 9 years

Objectives: Children will be able to:

1) Use fraction strips and fraction discs to show that smaller matching pieces can be put together to match a bigger piece;
2) Talk about this matching, *e. g.*, 2 quarters of something will match 1 half of the same thing;
3) Write about this matching in some acceptable way.

And childen will know that:

4) What they have found and written about are called *Equivalent Fractions*.

Instructional Materials:

1) 4 strips of paper or thin card for each child.
2) Copy of worksheet used in lesson on " What is a fraction", *i. e.* 6 oblongs, 6 squares, 6 circles drawn on paper.
3) Scissors, coloured pencils, paper and pencil, etc.
4) Teacher size strips and cutouts of oblongs, squares and circles.
5) Soft board and pins.

Learning Sequence:

Step 1: For home work previously, childen will have labelled, coloured and cutout 4 strips so that they have 1 whole strip, 2 halves (blue), 4 quarters (red), and 8 eighths (green).
"With your strips I want you to see how many ways you can match the pieces. Work in a group of 6. One of you act as recorder. You already know that 2 halves match 1 whole, see how many more matchings you can make. Will anything match $\frac{3}{4}$ for instance ? What about $\frac{2}{4}$?"

Step 2: Call child out to front to use teacher prepared strips. Group recorders share group findings with the class. As a matching is stated chosen child pins the teacher pieces on the soft board. As each new recorder speak class check to see if the matching is already shown, if not it is put on board.

Step 3: "You have shown that: $\frac{8}{8}$, $\frac{4}{4}$, $\frac{2}{2}$ match 1; $\frac{4}{4}$ match $\frac{2}{2}$; $\frac{8}{8}$ match $\frac{2}{2}$; $\frac{8}{8}$ match $\frac{4}{4}$; $\frac{1}{2}$ match $\frac{2}{4}$; $\frac{1}{2}$ match $\frac{4}{8}$; $\frac{2}{4}$ match $\frac{4}{8}$; $\frac{3}{4}$ match $\frac{6}{8}$. Would $\frac{1}{2}$ matches $\frac{1}{4}$ if the strips used had been of different sizes ?"

On-going evaluation: Give out worksheet on which are drawn the shapes. "You have found matchings using strips. Now see if the matching works for halves, quarters, eighths using the squares. Work in your groups again. Recorders report back as for Step 2.

Step 4: Now let us try a different fraction. Half the class use oblongs. You are group A. Your task is to see if you can fold your oblongs so that you have fifths. Leave one oblong unfolded. When you have folded fifths, try to make tenths. Label, colour and cut out pieces to show $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$ and see what matchings you can make with tenths. Group B, you are to fold your circles. Your task is to show thirds. Remember to leave one circle unfolded. When you have folded, labelled and coloured thirds, try to fold sixths. Can you find another fraction in that family. Colour and cut out your pieces to find as many matchings as possible."

Step 5: As for Step 2.

Step 6: "This renaming of fractions we have been doing has a special name. Does anyone know what we call it when we rename a fraction like $\frac{2}{5}$ as $\frac{4}{10}$? Yes, we call the *Equivalent Fractions.*"

Equivalent Fraction Game: A game for up to 4 players. You need: 1) A board (card paper) for each player. On the board is drawn 2 squares 20cm by 20cm; 2) Cutout of shapes, 2 full 20 × 20 squares, 8 halves (triangles), 8 halves (oblongs), 8 quarters (triangles), 8 quarters (oblongs), 16 eighths (oblongs); 3) A pack of small cards, on each is written one of these fractions. $\frac{4}{4}$, $\frac{4}{4}$, $\frac{8}{8}$, $\frac{8}{8}$, $\frac{6}{8}$, $\frac{6}{8}$, $\frac{4}{8}$, $\frac{4}{8}$, $\frac{2}{8}$, $\frac{2}{8}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{2}{2}$, making 20 cards in all.

To play:

1) Lay out shapes within reach of all. Give each player a board.
2) Shuffle cards and place in a pile face down.
3) In turn players pick up the top card and find a shape or shapes which match the fraction shown, *e. g.*, picks up $\frac{1}{4}$ may choose 3 quarters pieces or 6 eighth pieces. Player puts pieces on one of the squares on the board and puts card face down beside the pack.
4) First player to exactly fill both squares is the winner.
5) If a card is turned up that cannot be used. The player must miss the turn.

Key Topic 4 Operations on Fractions

Sub Topic 1 Addition of Fractions with Like Denominators

Class/Age Level: Grade 3; 8 years

Objectives: Children will be able to show with materials and numbers:

1) How to add fractions with like denominators of 2, 3, 4, 5 or 6;
2) How to retain the same denominator and just add the numerators;
3) That order is not important when you add fractions.

Instructional Materials:

1) Fraction strips used in lesson on equivalent fractions.
2) Fractional parts cut from oblongs, squares and circles used in lesson on equivalent fractions (See page 14.).
3) Fraction line cards marked: $\frac{1}{4}$'s, $\frac{1}{2}$'s, $\frac{1}{5}$'s, $\frac{1}{3}$'s, $\frac{1}{6}$'s, (See page 14.).

Learning Sequence:

Step 1: "We know about fractions: what they are, how to write them, how to rename them as equivalent fractions and how to put them in order. Can we add them ? Can we add whole numbers ? Does it seem likely that we can add fractions then ?"

Step 2: Make sure children have the materials listed 1 and 2 above. "In your group I want you to see if you can put pieces together to make new shapes. Let us look at halves together."
"Find a strip for one half. What are you starting with ? Now find another strip for one half. Remember it has to be from the same whole strip. Put your second strip alongside the first strip. How long is the whole strip. Can you tell me an addition sentence for what you did. Is this how you would write you just said ? $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$."

Step 3: "Now use your strips or shapes and see if you can make up some adding sentences about quarters/fourths and some about fifths. If you finish those you might use your circular pieces and see if you can make up some sentences about thirds or sixths."
Give the children time to manipulate the materials and write their own sentences.

Step 4: Write the sentences on the board as they are read out, *e. g.*, $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$.
"Who wrote those sentences ? Who wrote other sentences ?" Write them on the board or on a chart as the childen read them.

Step 5: "Look what we have here. Someone says $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ and someone else says $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$. Can they both be true ? Try it. Does it work like that with whole numbers ? Try $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$. Can we change the order here ?"

On-going evaluation: Point to a large number line showing halves and quarters."Who could come and show $\frac{1}{4} + \frac{2}{4}$ on this line" Repeat with other additions. "If we first show $\frac{1}{4}$ then another $\frac{2}{4}$, do we reach the same place as if we first showad $\frac{2}{4}$, then $\frac{1}{4}$" Repeat using number lines for fifths, for thirds, sixths.

Sub Topic 2 Addition of Fractions with Unlike, but Within the Same Family Denominators

Class/Age Level: Grade 4; 9 years

Objectives: Children will:

1) Know that if fractions have unlike denominators one has to find a common denominator before one can add them.
2) Know why one must find a commen denominator;
3) Know when one must find a common denominator;
4) Be able to add halves to quarters, fifths to tenths and thirds to sixths, to sums less than or equal to $\frac{2}{2}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{10}{10}$, $\frac{3}{3}$, $\frac{6}{6}$.
5) Know that order is not important when you add these fractions.

Instructional Materials:

1) Circular shapes marked in sixths and in thirds (See page 14.).
2) Fraction strips used in lesson on equivalent fractions.
3) Oblongs marked in fifths and tenths.

Learning Sequence:

Step 1: "Mother made 2 cakes of the same size. One she cut into 6 portions all the same size. The other she cut into only 3 matching pieces for hungry people. Dad took one of the big pieces. How much of that cake did he take ? Mum took one of the smaller pieces. How much of that cake did she take ? Indira was learning about fractions at school and she decided to work out how much of one whole cake Mum and Dad had eaten together."

Step 2: "Use your circular pieces to see if you can solve Indira's problem. What did you find ? $\frac{1}{3} + \frac{1}{6}$ is $\frac{3}{6}$. What denominators are there in that sentence ? Which is the bigger denominator ?"
"If Dad had been really hungry and eaten 2 of the big pieces, what fraction would he have eaten ? Find out using your shapes how much he and Mum would have eaten altogether in the case. What did you find ? $\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$. Are there still the same denominators as in the first sentence ? Which denominator appears in the sum ?"

Step 3: "Miri and Phil agreed to paint the fence. There were 20 boards in the fence. They thought it would take two days to do the job. Miri said, I will try to paint $\frac{2}{5}$ of the fence today. Phil said, I will try to paint $\frac{3}{10}$ of the fence today. How much of the fence would they have painted ?"
"Use your oblongs showing fifths and tenths to solve the problem. What did you find ? Did you write $\frac{2}{5} + \frac{3}{10} = \frac{7}{10}$?"

"Suppose Miri had worked extra hard and painted $\frac{3}{5}$, what would be the addition sentence ? Are the denominators the same as in the first sentence ? Is the bigger denominator used in the sum ?"

On-going evaluation: "When you learned about equivalent fractions you renamed in a special way. Who can give me an equivalent fraction for $\frac{1}{3}$, $\frac{2}{3}$? How about $\frac{2}{5}$, $\frac{3}{5}$?Let us write our addition sentences using these equivalent names. $\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$; $\frac{4}{6} + \frac{1}{6} = \frac{5}{6}$; $\frac{4}{10} + \frac{3}{10} = \frac{7}{10}$; $\frac{6}{10} + \frac{3}{10} = \frac{9}{10}$. Do we have the same sums ? What can you tell me about denominators when you want to add fractions ?"

Step 4: "Is it true that $\frac{2}{5} + \frac{3}{10} = \frac{3}{10} + \frac{2}{5}$? How do you know ?"

Sub Topic 3 Subtraction with Like Denominators

Class/Age Level: Grade 4; 9 years

Objectives: Children will show with materials and in writing:

1) How to subtract fractions with like denominators of 2, 3, 4, 5 or 6;
2) That subtracting and adding fractions belong together in the same way as subtracting and adding whole numbers.

Instructional Materials: As for lessons on adding fractions with unlike denominators (Sub Topic 2).

Learning Sequence:

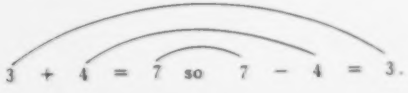
Step 1: "Let us think about the cake that Mum cut into small pieces (Sub Topic 2). She cut 6 matching pieces so how much of the cake was each piece ? How many sixths in the whole cake ? Use your shapes to show how much of the cake is left after Mum eats one piece. If you are taking something away, what sign do you use ? Tell me the subtraction sentence about the one piece Mum ate. Do I write it like this $\frac{6}{6} - \frac{1}{6} = \frac{5}{6}$?"

Step 2: "Now in your group see how many sentences you can show and write about taking away pieces of the cake. Don't forget that you might take a piece when there was only $\frac{3}{6}$ left. Don't forget that some people might be hungry and eat more than one piece."

Step 3: Recorders share with class the sentences they have shown. Write on board or chart as they the sentences.

On-going evaluation: Repeat Steps 1 to 3 using the story about painting the fence, *e. g.*, if Miri and Phil together painted $\frac{7}{10}$ in one day, how much of the fence remains to be painted ?

Step 4: "When we used whole numbers we found that adding and subtracting belong together in a special way and we used the rainbow to show it.



Do you think this would work when we use fractions ? Try it. Try $\frac{3}{5} + \frac{1}{5}$. Did it work ? Is it true that if $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$ then $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$. Check by drawing in the rainbows."

Sub Topic 4 Subtraction with Unlike Denominators

Class/Age Level: Grade 4; 9 years

Objectives: Children will show with materials and in writing how to

choose a common denominator when subtracting quarters from halves, and tenths from fifths.

Instructional Materials:

1) Firm fruit like apples. Knives to cut.
2) Square shapes masked in $\frac{1}{2}$'s, $\frac{1}{4}$'s.
3) Oblong shapes masked in $\frac{1}{5}$'s, $\frac{1}{10}$'s.

Learning Sequence:

Step 1: "I have an apple here and I want to eat half of it myself. How much will be left ? My friend says she would like some apples. If I give her a piece the size of $\frac{1}{4}$ of the whole apple, how much of the apple will be left for someone else ?"
Ask a child to come and cut one apple into 2 halves. Set aside the piece for you.
"How many quarters in the whole apple ? Who thinks they could cut a piece the size of $\frac{1}{4}$ from this half piece ? How much of the apple is left ? Is it the same size as the piece you have just cut ?"

Step 2: "With your square shapes show some take-away stories. Try $\frac{3}{4} - \frac{1}{2}$. When you show a take-away write the subtraction sentence. Share your findings with your group."

Step 3: Have children report on their findings. Write their sentences on the board or a chart.
"What denominators were used ? Is the bigger one in the difference ?"

Step 4: Use the problem of painting the fence again.
"When Miri and Phil had painted $\frac{4}{5}$ of the fence, how much more was this than when Phil had painted $\frac{3}{10}$? What sentence did you write ? Is it true that $\frac{4}{5} - \frac{3}{10} = \frac{5}{10}$? Is the bigger denominator still used in the difference ?"

Step 5: "When you added fractions with unlike denominators, what did you do ? Can you name an equivalent fraction for $\frac{4}{5}$ to make it possible to take $\frac{3}{10}$ away ?"
Give more similar examples.

On-going evaluation: "What must you do when you have to add or subtract fractions with unlike denominators ?"

Key Topic 5 Concept of Place Value to the Right of the Decimal Point and Operations on Numbers with up to 2 Decimal Places

Sub Topic 1 Introduction to Decimal Fractions through the Use of Place Value

Class/Age Level: Grade 4; 9 years

Objectives: Children will know and be able to show with materials:

1) That it is not possible to divide a single digit whole number exactly by 10. One has to move into a new set of numbers;
2) That $5 \div 10$ can be written as 0.5, $1 \div 10$ as 0.1, $6 \div 10$ as 0.6, etc. and these are called decimal fractions;
3) That 0.5 means no ones and five tenths;
4) That the dot • is called the decimal point;
5) That digits move into different places in relation to the decimal point but the point retains its position.

And children will be able to:

6) Name the face, place and total value of any digit in a number with places to the left and one place to the right of the decimal point.

Instructional Materials:

1) Place value blocks or replicas in card or paper of place value blocks (See page 13.).
2) Abacus or replicas in clay, straws, banana leaves, sticks, etc. (See page 14.).
3) Place value cards with digit faces (See page 14.).
4) Newspaper, wrappers from cans, packets.

Learning Sequence:

Step 1: Show the children a clipping, wrapper or packet on which is written a numeral with digits to the left and right of the decimal point. Draw a frame around the numeral.
"Who can read what is shown in the frame ?"
Accept all responses without comment. On the black board write: Clearance of this bridge is 5.5 metres.
"Who can read sentence ? What does it mean ? What can you tell about the face value of the 5's ?"

Step 2: Show the place value cards and digit faces.
"Find me a face showing 5. Put it in the 100's place. What is its total value now ? Put it in a place where it will be worth 10 times smaller. Where is it ? If you divide 500 by 10 what do you get ?"
"Now move the 5 so that it is 10 times smaller than 50. Where is it ? What is the total value now ? Is there anywhere to show the number 10 times smaller than 5 ? What is $5 \div 10$?"
Remind the children that they got another board when they wanted to show a number 10 times bigger than 500.
"What could we do now we have no more places going to the right ? Yes ! We get another board."
Place the new board in position.

"What is drawn that links the ones period board and this one ? Who knows what we call that dot or spot ? We call it the decimal point."

Step 3: "Look at the new board. What is the same about it ? What is different ?"
Draw attention to the pattern H. T. O. which is on all boards and to the special headings Tenths, Hundredths, Thousandths on the new one.
"Show me the number 10 times smaller than 5. What place is it in ? We would write it 0.5. What does it mean ? There are no ones and 5 tenths. Do we write 0 in front of whole numbers ? Why not ?"

Step 4: "Who can show on the boards the number that was on the newspaper clipping or wrapper ? Do we know what it all means yet ? Do we know what the digit to the left of the decimal point and the first on the right of it means ?"

On-going evaluation: *Either* "Show 5.5 on the abacus. Which prongs have you used ? Show the number 10 times bigger. What is the total value of the number shown now ? What is $55 \div 10$? Show it on the abacus. Write it. Read it." *or*, "Let the long block represent one. Show 5.5 with blocks. What shapes have you used ? Show the number 10 times bigger. What shapes have you used ? What number have you shown ? What is $55 \div 10$? Show it with blocks. Write it. Read it."

Step 5: "Write on the board 63.2. Tell me the face, place and total values of each digit. Is the .2 a numeral for a whole number ? Is it bigger or smaller than the whole number one ? We say that 0.2 is a decimal fraction. Show the decimal fraction 0.3 on your material. Now show the decimal fraction 0.9."

Sub Topic 2 Addition with 2 Decimal Places

Class/Age Level: Grade 4; 9 years

Objectives: Children will:

1) Know that when you add numbers in decimal places it is just like adding whole numbers;
2) Be able to rename (regroup) hundredths as tenths and tenths as ones;
3) Be able to apply this knowledge to solving everyday word problems involving money or some other suitable measurement.

Instructional Materials:

1) Place value blocks or replicas (See page 13.).
2) Abacus or replica (See page 14.).

3) Coins and notes or weights or capacity jars as appropriate.

Learning Sequence:

Step 1: "We know how to add whole numbers and we know about and can read and write numbers with decimal places. Is it possible to add numbers with decimal places ? If it is, do we have to do anything different from when we add whole numbers ? Who can think of a number with 2 decimal places ? Tell me how to write it. Show it with your blocks. Let the flat stand for one. Now who can think of another number with 2 decimal places that they would like to add to the first one ? Tell me how to write your number. Show it with your blocks."

Step 2: "If these were whole numbers what would you do with the blocks ? Try it with the blocks that are showing mixed numbers. Tell me your example,

e. g. 4.32
+5.64

Would it help if we labelled the columns ? Do we need to ?"
Note: Some children find it helpful, others don't need it.
"Did the decimal point move ? Did you have to rename (regroup) in this example ?"

Step 3: "Now let us try my example. All show and write my first number which is 3.88. I want to add 4.55 to that number. Show and write my second number. Now find the sum. What did you do with 13 little cubes ? What did you do with the 14 longs ? What number is in the ones place in your sum ? Was it just like adding whole numbers except there is a decimal point ?"

Step 4: "Will it be true that $3.88+4.55=4.55+3.88$? Prove it using two sets of abacus.

On-going evaluation: "Who can think of an everyday situation where numbers would be in decimal places ? Try Johari's example. He bought a record costing $7.50 and another which was more expensive. It cost $9.75. How much did he spend altogether. Use blocks or notes and coins. Now try Ubol's example. She has 4.75 litres of orange juice and 3.75 litres of mango juice. She knows she will need at least 8 litres of juice for her party. Will she have enough ? Use blocks or capacity jars."

Sub Topic 3 Subtraction with 2 Decimal Places

Class/Age Level: Grade 4; 9 years

Objectives: Children will:

1) Know that when one subtracts numbers in decimal places it is just like subtracting whole numbers;

2) Be able to rename ones as tenths and tenths as hundredths;
3) Be able to apply this knowledge to solving everyday word problems involving some suitable measurement;
4) Be able to write a related addition sentence for a subtraction example.

Instructional Materials:

1) Place value blocks or replicas (See page 13.).
2) Abacus or replicas(See page 14.).

Learning Sequence:

Step 1: "We know that adding decimals is like adding whole numbers except that we have a decimal point. Do you suppose that subtracting decimals might be like subtracting whole numbers ? How could we find out ?"

Step 2: "What numbers did you think of ?

Example without renaming

O	T	H
9 .	7	6
−4 .	5	3
5 .	2	3

Example with renaming

O	T	H
~~9~~ 8 .	~~4~~ 13	~~2~~ 12
−5 .	8	4
3 .	5	8

"Did you do anything different from what you do with whole numbers ? What did you do to make it possible to take 4 hundredths from 2 hundredths ? 8 tenths from 3 tenths ?"

Step 3: Give more examples for practice. Some children could use the abacus, some the blocks. Leave the column headings in at this stage.

Step 4: "How do we give change ? Does the store keeper write down something like $8.45 Why not ? If you spent $3.96 what would
−3.96
you most likely use to pay for the goods ? Can you think of an everyday situation where you would have to write a subtraction like that with numbers in decimal places. Think about it at home tonight and we'll talk about it tomorrow."

Note: It is quite difficult to find realistic situations within a child's experience where subtraction with decimal places would be needed but given time they might think of problems something like: I saw a dress in a window and it cost $19.75. I had saved $14.50 (or $14.98) out of my allowance. How much more would I need ?

Step 5: "When we subtract whole numbers we are able to write a related addition using the rainbow to check. Can we do this when we subtract numbers with decimal places ? Try it. Is it true that if $8.45-3.96=4.49$ then $4.49+3.96=8.45$? Show it on the abacus."

Sub Topic 4 Multiplication of Decimal by Whole Number from 2 to 9

Class/Age Level: Grade 5; 10 years

Objectives: Children will be able to:

1) Show the multiplication process by use of materials;
2) Multiply decimal fractions by one-digit numbers;
3) Solve word problems involving multiplying a one place decimal by a single digit.

Instructional Materials:

1) Environmental materials such as containers, icecream sticks/ bamboo, etc.
2) Place value blocks or replica (See page 13.).
3) Number cards including symbols or sentence card (See page 15.).

Learning Sequence:

Step 1: A: Using 10 bottles or containers

"Here we have 4 bottles of juice (coloured water). In each bottle, there is 0.2 litre of juice. What would we do if we want to take the juice home ? Do you want to carry all the bottles ? If we put the juice in one container, how much will those be ?"

Children pour the juice into a different container.

"How do you get the answer ? We measure them/we add $0.2+0.2+0.2+0.2$/or we multiply 0.2 by 4. How many bottles are there ? How much water in each bottle ? So, how many times of 0.2 ? Then, what do you think is the simplest way to find the answer ?"

B: Using 10 sticks of equal length.

The procedure is the same as in A except the measurement is in metres. Children place the sticks end to end and find the total length through multiplication.

C: Using place value blocks.

The procedure is the same as in A. With this material we have to use groups of tenths (using the small cubes). Children group all the tenths and give the total number of tenths through multiplication.

Step 2: Children use number cards 2 to 9, 0.2 to 0.9 and symbols $\times$, $=$.

"For each of the examples or situations show me a

multiplication sentence using sentence cards."

Children place the number cards and symbols to form a sentence.

Example (based on Step 1, A): | 0.2 | × | 4 | = | 0.8 |

"If you multiply tenths by a whole number what do you get ?"

On-going evaluation: Children practice using work cards.

Examples

1. $0.2 \times 3 =$
2. $0.6 \times 4 =$
3. $0.8 \times 2 =$
4. $0.9 \times 6 =$
5. $0.8 \times 7 =$
6. $0.7 \times 5 =$
7. $0.4 \times 9 =$

Step 4: $0.5 \times 8 = \square$

"See if you can make up a story in words to fit this problem."

Key Topic 6 *Measurement*

Sub Topic 1 Length and Distance

Class/Age Level: Grade 3; 8 years

Objectives: Children will be able to:

1) Measure and compare lengths using unconventional units;
2) Explain the necessity for standard units (metre and centimetre);
3) Use metre and centimetre in measuring length and distance;
4) State the relationship between metre and centimetre;
5) To record the length and distance measured in metre and centimetre in words.

Instructional Materials:

1) Environmentral materials such as wooden strips, sticks/leaves/cardboards, rope/twine/wool/strips of cloth.
2) Elements of the body: fingers, elbow, arms, foot, height.
3) Rulers: one-meter, 30cm.
4) Measuring tape (metric system).

Learning Sequence:

Step 1: Choose 2 objects which can't be shifted such as doors, windows, etc.

"How would we measure them to see which is longer ? We do not have a ruler or a tape measure. What shall we use ?"

Children use different unconventional units and report their findings to the class.

"Why did those using the whole body get a smaller number than those using a span ?"

Step 2: Give children a piece of rope or string which is 1 m long. Don't state actual length. Repeat measuring done in Step 1.
"Why did we all get the same number ?"

Step 3: Give children a metre rule. Choose 10 objects to be measured. Ask children to complete a chart like the one shown as they work.

Name of object	Just, Less than, More than, Equal to 1 metre?
My guess	
Actual	
My guess	
Actual	

Step 4: Repeat Step 3 with children measuring objects outside the classroom and/or in their homes.

Step 5: "Look at the metre stick carefully. How many longish lines are shown ? Are these intervals shown on your school ruler ? How many are there ? Who knows what we call one of these intervals ? How many centimetres in the whole metre ?"

Step 6: Children measure smaller objects using centimetre as the unit, and completing the chart shown as they work.

Name of object		Length in cm
	My guess	
	Actual	
	My guess	
	Actual	
	My guess	
	Actual	

Step 7: Give children a metre rule and a piece of string or wool. Assign

groups to measure each around the other neck, head, waist, etc. Get them to complete the chart shown as they work. They use string to measure, then measure the string against the rule.

Items	Name of person	Distance round
Head Neck Waist		
Head Neck Waist		

Step 8: Give children a metre rule. Choose objects which are suitable for measuring in metres or in centimetres.
"As you measure, complete the chart shown."

Item	Estimate		Measure		Difference	
	m	cm	m	cm	m	cm

On-going evaluation: Show children several objects and ask them to state which is the more suitable unit of measure to use metre or centimetre. Get them to make a list of objects that they would measure in metres, then draw pictures of the objects. Then make a list of objects that they would measure in centimetres, and draw pictures of these objects.

Key Topic 7 *Plane Geometric Figures*

Sub Topic **Perpendicular Lines**

Class/Age Level: Grade 5; 10 years
Objectives: Children will be able to:

1) Identify a square corner/right angle;
2) Fold a rough torn paper to make a sqaure/right angle;
3) Use their folded paper to test corners in the room to see if they are square;
4) Identify places outside the classroom where such things as roads, buildings, footpaths meet to form a right angle;
5) Show that right angles can have different orientation;
6) State that when two straight lines meet or intersect and form a

right angle, then they are said to be perpendicular.

Instructional Materials:

1) Roughly torn pieces of paper.
2) Match sticks.
3) Plasticine or clay.

Learning Sequence:

Step 1: Give each child four match sticks. Form a square with the sticks.
"How do you know it is a square ? What can you find at one corner of the square ? What can you say about the two sticks that form the square corner for right angle ? Can you identify objects around you which have square corners ?"

Step 2: Place a strip of plasticine/clay on the table. Give each child a match stick.
"You are the match stick. Show me how you would stand on the ground (Plasticine represents the ground). Are you all standing straight up ? Let us see who is not."
"Do you make a square corner or right angle with the ground ? Can you show me a situation where you are not making this angle with the ground ?"

Step 3: "How could we confirm that you are really making a square corner ? How do we measure a right angle ? If you have only a roughly torn piece of paper (give each child one), how would you fold it to form a square corner or a right angle ? Are you sure you have made a square corner ? Test it by matching it with the corner on your exercise book."

Step 4: "Use your folded corner to test the corners in the classroom. Make a list of the objects which have square corners."

Step 5: "Unfold the square you have made. Draw lines along the folds. Do the two lines meet to form a right angle ? What can you say about the two lines ? Does anyone know what we say about the place where two lines meet or intersect to form right angles ? Yes, they are said to be perpendicular."

Step 6: Ask children to fold paper to obtain two lines perpendicular to each other. Then paste them in their exercise books.
"If we turn the paper a little, are the lines still at right angles ? Are they still perpendicular each other ?"

"Show on a clock face times when the hands are perpendicular to each other. Are they perpendicular at 1:25 ? What other times ?"

On-going evaluation: "Now can you find some places outside the classroom where such things as roads, buildings, footpaths meet to form a right angle ? Draw some of them."

Problem Solving

Introduction

While in all the examplars a problem solving approach is used, the major focus of each learning experience that follows is directed to problem solving. The particular content, *e.g.*, addition of whole numbers, measurement of weight or capacity, is less important than are the mathematics processes to be developed, however both must be within the capability of the children.

The lessons may be scheduled at any time if these conditions are satisfied. Each lesson shown here is designed to focus on the actions of the teacher in promoting these processes.

Ideally there should be no need for lessons such as these because all of the objectives would be incorporated into the *content oriented* topics such as those described elsewhere in this chapter. However it is generally conceded that problem solving is presently under-emphasized in most classrooms.

In examining lessons such as these desirable behaviours can be observed and be assimilated into the broader curriculum. In addition, these lessons demonstrate that problem solving can be stimulated by the use of concrete materials.

Sub Topic 1 Mathematical Relations Hidden in a Concrete Situation

A rectangular vessel partly filled with water is familiar to the children, and there are many similar vessels in our living circumstances. Children will be able to find various kinds of mathematical relations by tilting it. They must be able to describe, and defend these descriptions. The use of mathematical terms and symbols to communicate clearly is a desired objective of mathematics education, and is coupled here with some open-ended tasks which require the children to develop their own conjectures and test them.

Class/Age Level: Grade 6; 11 years

Objectives: Children will be able to:

1) Observe and describe the different shapes and sizes of the water

surface;

2) Establish the mathematical relations hidden in such a concrete situation;
3) Form conjectures and translate them into symbolic form;
4) Test the conjectures.

Instructional Materials:

1) Rectangular vessles with scale 6 cm x 10 cm x 15 cm (See page 13.). One per child, or at least one per group of up to 8 children.
2) Beakers, 1 litre (See page 13.).
3) Water.
4) Worksheets.

Learning Sequence 1:

Step 1: "Look at these rectangular vessels and take one each and fill it half with water. We can tilt this vessel in different ways such as these."
Demonstration but always restricted to keeping an edge on the table.

Step 2: "The problem is to find as many relations as possible between the sides or shapes or sizes when we fix a side of the base and tilt it at various angles. (Demonstration) Write your observation on the sheet."
Some children's conjectures that may be offered are: 1) Length of the sides varies; 2) Area of surface varies; 3) Width of water surface increases; 4) As length of a side increases, the length of another side decreases; 5) Area of a surface increases; 6) Surface is horizontal; 7) Volume doesn't vary; 8) Shapes of the base and surface are rectangles; 9) Shapes of the sides vary from trapezoid to triangle; 10) Length of a side cannot increase beyond 15 cm; 11) Sum of "a" and "b" is constant; 12) A fixed point exists when we look from the side.

Step 3: "Now form a group of up to eight children and discuss your observations Choose the reporter to prepare the list of relationships you have observed.

Learning Sequence 2:

Step 1: Reporters of groups present their observations.

Step 2: "We have many observations. It is difficult to remember all these. Many of them are similar. Try to classify them on the basis of similarity."
Classification of conjectures the children might offer (Figures of the right side show relevant conjectures appeared in Step 2 above):

1) Sum is constant·····11)

2) Variation ·····1) 3) 4)
3) Domain·····10)
4) Shape of water surface ·····8) 9)
5) Area ·····2) 5)
6) Volume ·····7)
7) Others ·····6) 12)

Step 3: "You found that the sum of side "a" and side "b" is constant. Can you explain how you arrived at this conjecture?"

Step 4: "Can you explain any other observations? Tomorrow, you could explain and test your observations from the various viewpoints, and see if you can write about what you found out."

Sub Topic 2 Triangles

Note: It is assumed the children know that an isoceles has two matching sides.

Class/Age Level: Grade 3; 8 years.

Objectives: Children will be able to:

1) On the geoboard make at least one isosceles triangle using a given fixed line:
2) Test whether a triangle made is an isosceles triangle or not;
3) Make up another similar problem.

Instructional Materials:

1) Tack-board arranged 6×6 nails or geo-board 5×5 nails (See page 16.).
2) Coloured rubber bands.
3) Worksheets, consisting of four representations of the geo-board or tack-board.

Learning Sequence 1:

Step 1: "Look at these tack-boards and coloured rubber bands. Each take some rubber bands. A piece of tape is already attached to each board to represent a fixed side of the triangle. We could make several figures using bands on the boards. Today, please make isosceles triangles using the fixed line on the geo-board as one side. For each isosceles triangle you make, draw a picture of it on your worksheet."

Step 2: Each child or group displays the drawings to the rest of the class.

Step 3: "Test whether your triangles are isosceles or not, and present your observation."
Children will test them by using rulers or compasses.

Learning Sequence 2:

Step 1: "Make up another problem similar to the first one."
Teacher encourages children to make up problems freely.
Children's problems that might be made up:

1) Children will move the fixed side.
2) Children will change the isosceles triangle to another kind of triangle.
3) Children will change the isosceles triangle to another plane figure.
4) Children will change the shape of geo-board.
5) Children will change the size of geo-board.

Step 2: "Present your problems to the rest of the class."

Step 3: "We have many problems. But some problems are similar. Try to classify them and explain which parts of problems are similar."
Children will classify and explain.

On-going evaluation: Choose one of the problems written by children. "Let us all see if we can solve this problem."

Sub Topic 3 Manipulating Numbers

Class/Age Level: Grade 5; 10 years

Objectives: Children will be able to form hypotheses about:

1) Measuring using different sized weights;
2) Different sets of coins;
3) Test their hypotheses.

Instructional Materials:

1) Weights for 1, 2, 3,·····10 grams.
2) Coins and notes or replicas
3) Worksheet showing:
   a) Here is the problem to solve. Shigi has 4 currency notes. He can pay each amount $1 through to $10 without getting change. What notes did he have?
   b) Norma has 100 one yen coins. She has placed them in seven envelopes. She can pay from ¥1 to ¥100 by using one or many envelopes but not opening any of them. Can you find the amount in each envelope?

Learning Sequence:

Step 1: "Here are 4 weights: 1g, 2g, 2g, 5g. What is the greatest amount of baking powder you could weigh out by using these weights? You may only put powder in the pan once. Will 5g be the greatest amount? Why not? What might you do to the numbers to find the greatest amount? Why did you say add?"

Step 2: "In your group, see how many of these amounts you could

weigh out using only the 4 weights given. Can you weight 1g, 2g, 3g,·····10g?"

Step 3: Children report back.
"How did you decide to weigh 6g, 9g? What operation on numbers did you use?"

Step 4: "We have a new problem. You may use any weights 1g, 2g, ·····10g and you may use any of them as many times as you like, but you may only list 4 weights each time. With this limitation see how many ways you would weight out 10g. For instance you might decide to use 1g, 1g, 1g, and 7g. Make a table showing the combinations you choose. What operations on numbers did you use each time?"

Step 5: "Now see if you can solve this problem. You can use any of the weights 1g, 1g, 2g, 6g to see how many quantities from 1g to 10g you can weigh out directly, and indirectly. Make a table."
"When you measured directly what did you do to the numbers? When you measured indirectly by putting a weigh in the pan along with the baking powder what did you do to the numbers?"

On-going evaluation: Assign the problems on the worksheets.

Sub Topic 4 Addition Facts for 6

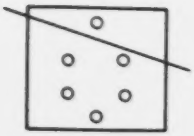
Class/Age Level: Grade 1; 6 years
Objectives: Children will find all the addition facts for 6.
Instructional Materials:

1) 15 cards.

2) 100 pebbles.
3) 15 hard cardboard strips.

Learning Sequence:

Step 1: "Here are pebbles and cards."
One card is placed on the table.
"I am placing pebbles at the points on the card. By placing hard cardboard sheet, it can be divided into two parts as you can see. How many pebbles are on this side? (Pointed by teacher.) How many on other side? (After removing hard board strip.) How many pebbles are there in all? How would we write that as an adding sentence?"



Step 2: Form groups of four. Each group will take one card, 6 pebbles and a hardboard strip.
"Put the pebbles on the points marked in a card. Your task is to find out in how many different ways can you divide up the stones. Write about each pattern you make like this: 5+1=6"

Step 3: "Kamol, can you give me your observations?"
Teacher writes the Kamol's observations.

$$1+5=6;\quad 3+3=6;\quad 2+4=6$$
$$5+1=6;\quad 4+2=6$$

"Who else has written what Kamol said? All of you compare your results."

Step 4: "Which of your sentences is different from the rest? Do any others have a 3 in them? Are there any patterns in your sentences?"

On-going evaluation: "Now you put one more pebble on your card. Remember it should not touch any of the previously placed pebbles. Do the same activity and find the different ways of dividing them up. Make a list of your findings. Try to find some patterns in your sentences."

Step 5: Report back and discuss the findings.

Evaluation

Evaluation needs to be made in two main ways: first, as immediate feedback from the activities generated within the lessons. Such evaluation will be related to the specific objectives of the lesson. Second, long term evaluation which will relate to the objectives of the whole programme.

Observation during the lesson of the effective use of problem solving strategies to get started, and to overcome potential blocks can be supplemented by a more formal evaluation of the output of individual children or groups. The progress made and quality of the partial or total solutions, and the variety of them, can be noted and valued.

Long term evaluation poses greater difficulty, since strategies and attitudes develop slowly and are formed by the interaction of the child with the whole curriculum. Once a child has solved a problem, or has acquired skill in using an algorithm for its solution, further encounters with it may well be exercises involving mainly skill and recall. The critical

test of the development of problem solving ability is the capability of the individual to solve problems. Therefore the curriculum must provide such opportunities regularly.

Chapter IV Some Strategies to Encourage the Use of Instructional Materials

Introduction

The concern expressed in the previous report, that ". . . the use of textbooks has been dominating the teaching and learning of mathematics" is justified. Despite the research evidence such as that reported by Jacobsen and Sinha, which provides compelling support for the use of a wide range of concrete instructional materials, there are powerful factors which work against the regular use of such materials in the classrooms. Although this applies particularly to individual children's use, it also inhibits even their use in teacher demonstration.

When teaching for problem solving and teaching through concrete materials are both advocated, a second set of factors might act in concert to reduce further the prospect of change. However, both dimensions require urgent attention and it is possible to address both issues simultaneously. The strategies enumerated later involve both dimensions, sometimes explicitly, but always implicitly, as also illustrated previously in the implicit and explicit approaches to problem solving used in the exemplar lessons in Chapter III.

If exemplary lessons such as those described in the previous chapter are to be more applicable and useful throughout the region, then these factors must be identified so that appropriate strategies may be proposed which would remove their effect and promote more desirable modes of teaching and learning.

Target Groups

In identifying these problems and strategies, the participants of the workshop recognized that there are many bodies and individuals who have responsibilities in this area. They range from national governments to individuals, and a range of actions is proposed, where appropriate, for each problem and each body. Such target groups include:

1) Schools and individual teachers;
2) Associations of teachers;
3) Consultants/Advisers;

4) Researchers;
5) Teacher training institutions;
6) Government and administrative agencies;
7) Regional cooperative agencies; and
8) Community groups.

Problems Identified

The major problems identified at the workshop fall into three broad categories, and these problems may vary from country to country.

A : Those concerning the teacher;
B : Those concerning the supply of instructional materials; and
C : Conditions for teaching mathematics.

The list which follows is cumulative. Only some of these apply in any particular country.

A. Teacher Related Problems

1) The attitude of the teachers to the use of concrete materials and the problem solving approach.
2) They still need more knowledge in mathematics.
3) They are inexperienced about what materials to use and how to use them.
4) They are not yet convinced that children should handle materials themselves rather than just watch a demonstration.
5) They are inexperienced in the use of materials when evaluating or assessing children's performance and attitude.

B. Provision of Suitable Instructional Materials

Problems under this category include:

1) The high cost of commercial materials and their lack of availability in some areas which militate against their ready use.
2) The teachers are unfamiliar with the ways in which such materials can be replicated at low cost.
3) The lack of suitable centres where, under guidance, teachers can make these materials.
4) Some government agencies are not yet convinced of the need to stimulate the supply of such materials, and to update printed resources.
5) Everyday objects are not collected and made available to children in sufficient quantity and variety.

C. Conditions for the Teaching Mathematics

Major problems identified under this heading are:

1) Class size is in general too large for effective use of the problem solving approach and concrete materials.

2) Lack of storage for materials and lack of tools to construct the materials.
3) Insufficient time for teachers to prepare and to collect such materials.
4) Classrooms are often not organized to promote the effective use of materials.
5) Need for more encouragement from school authorities and peers to use this approach.

The following table shows the interrelation between the problems and target groups identified. By degree these relationships may vary from country to country.

Problems		Target Groups							
		1	2	3	4	5	6	7	8
A	1	×	×	×	×	×	×	×	
	2	×	×	×	×	×	×		
	3	×	×	×	×	×	×	×	
	4	×	×	×	×	×	×		×
	5	×	×	×	×	×	×		×
B	1	×	×	×			×		×
	2	×	×	×		×		×	×
	3	×	×	×		×	×		
	4						×		×
	5	×	×			×			×
C	1	×	×	×	×	×	×		
	2	×	×				×		×
	3	×		×			×		
	4						×		
	5	×	×	×	×	×	×	×	

From this table it is quite clear that in finding solutions to the problems, not only must schools and teachers be involved but all agencies to some degree. The onus falls especially on government departments, consultants/advisers, teacher training institutions and associations of teachers.

These agencies have responsibility not only to help overcome the problems identified but also some of them have to act as advocates to the general public and teachers. They must create a climate of opinion about the utilization of the problem solving approach and concrete materials which will in turn affect their supply and use.

Strategies

During the discussion many strategies for overcoming the problems were

identified and it was agreed that some of them would be applied to several of the target groups.

The following table shows these common strategies which concern

Strategies	Target Groups							
	1	2	3	4	5	6	7	8
1. Providing more knowledge in mathematical content in pre-service and in-service training.	S ×	×	×		×		×	
2. Providing more and regular opportunities for teachers to learn about the need for and ways of using concrete materials.	S ×	×	×		×	×	×	
3. Creating more awareness of the need for using the problem solving approach.	S ×	×	×	×	×	×	×	×
4. Providing more research into the needs of teachers and the school in which they work if they are to use this approach.	S ×	×	×	×	×	×	×	
5. Conducting classes for parents through such organizations as PTA's.	S/T ×	×	×			×		×
6. Providing more intensive training to skilled teachers to become resource persons.	S ×		×		×		×	
7. Giving more training on the development and use of materials to the teacher-educators.			×		×	×	×	
8. Encouraging and supporting innovative teachers to try out new ideas.	S ×	×	×	×	×	×		
9. Using a multi-media apppoach when providing these learning opportunities at all levels.	S/T ×	×	×		×	×	×	
10. Providing schools with funds to purchase materials or providing them with the materials.						×		×
11. Providing resource centres/areas where teachers and student-teachers can make materials.	S ×	×			×	×		
12. Using community help to make materials.	S/T ×		×					×

Strategies	Target Groups							
	1	2	3	4	5	6	7	8
13. Updating printed materials for use by children.			×	×	×	×		
14. Supplying printed materials to use by teachers to help them to make and understand the use of materials.		×	×		×	×		
15. Subscribing to professional journals on the teaching of mathematics.	S ×	×	×	×	×			
16. Using trained resource teachers to help others in nearby schools.	T ×		×			×	×	×
17. Gathering teachers together from clusters of schools to share new ideas and materials.	S ×	×	×		×	×		
18. Taking part in pilot schemes and their evaluation.	S/T ×	×	×	×	×	×	×	×
19. Evaluating the effects of using the problem solving approach and concrete materials.	S ×	×	×	×	×	×	×	×
20. Recognizing elementary school teachers as professionals.	S ×	×	×	×	×	×	×	×

Note: For figures under Target Groups, please refer to the relevant section.
S stands for School and T stands for Teacher.

learning/training, resource/materials, cooperation/communication, policy -facility and the target groups they would be used on.

Many of the strategies listed above are already in use in some countries but their effectiveness nonetheless requires constant monitoring. Any strategy which is proposed to use should be carefully examined to ensure that it will not create other undesirable and unnecessary stress and that it is the most appropriate one to use in the particular circumstance.

In every step of the strategies, constant monitoring and feedback are required to ensure effectiveness of the strategies as well as to provide information which might be needed for the modification of strategies. The mechanism of monitoring and feedback could be done through questionnaires, observations, dialogues, or other effective and reliable means.

A coordinated approach is more likely to produce a significant change, since a mix of strategies will be needed in most circumstances, and priorities may need to be established. The source of coordination is likely to vary according to the relationships between the agencies in the system.

The setting up, where none exists at present, of curriculum agency whose function is the development, implementation and monitoring of the effect of the use of resources will help to overcome many of the identified problems.

There are some problems that only government agencies and teacher organizations who negotiate with them can overcome. These problems are those of class size and the provision of suitable classrooms and storage areas.

In addressing the issue of how to use the materials to develop problem solving, participants became aware of the additional difficulties to be overcome if problem solving is to be accorded the priority in elementary matheamtics education it deserves. Accordingly it is recommended that, in order to explore problem solving in elementary mathematics, participants urge Unesco or other relevant regional institutions to initiate a joint cooperative programme to examine the problems and issues hidden in teaching of problem solving and to develop strategies to overcome these problem solving.

Appendix I List of Participants

Participants

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Ms. Hanako Senuma	Researcher Ditto.

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Mr. Satoshi Kawanobe	Director, External Service Department
Mr. Toshio Kanaya	Chief, Section for Educational Co-operation in Asia, External Service Department
Mr. Miharu Kajita	Senior Researcher, Ditto.
Mr. Nobuya Higuchi	Researcher, Ditto.
Mr. Ryo Watanabe	Researcher, Ditto.
Ms. Kumiko Sunakawa	Assistant, Ditto.
Mr. Morito Kanegae	Associate fellow attached to the Section for Educational Co-operation in Asia

Appendix II Membership of the Working Groups

I Members of the Working Group for the Examination of Instructional Materials Brought by Participants

Group A
- Mr R J Gaffney, Australia
- Mr S Motin, Bangladesh
- Mr H N Gupta, India
- Dr D S Naga, Indonesia — Rapporteur
- Mr E Nagasaki, Japan
- Ms I Castro, Philippines — Chairperson
- Ms M L Goh, Singapore
- Mr P T Wijetunga, Sri Lanka

Group B
- Mr F Cao, China
- Ms H Senuma, Japan
- Mr N P Sharma, Nepal
- Mr S I M Bokhari, Pakistan — Chairperson
- Mr E Sikil, Papua New Guinea
- Mr D Teo, Singapore — Rapporteur
- Dr Piyavadee W, Thailand
- Dr Ed Jacobsen, Unesco

Group C
- Mr R Chand, Fiji — Chairperson
- Dr D K Sinha, India
- Mr M Khairuddin, Malaysia — Rapporteur
- Ms J Paske, New Zealand
- Mr O K Kang, Korea
- Mr K S Gan, Singapore
- Dr G C Nguyen, Viet Nam

II Members of the Working Group for the Development of Exemplars

Group A
- Topic 1:
  - Mr S Mothin, Bangladesh
  - Mr E Sikil, Papua New Guinea
  - Dr Piyavadee W, Thailand
- Topic 2:
  - Ms I Castro, Philippines
  - Mr O K Kang, Korea
  - Mr K S Gan, Singapore

Group B
- Topic 3: Mr F Cao, China

Ms M L Goh, Singapore

Topic 4: Mr R Chand, Fiji
Mr D Teo, Singapore

Topic 5: Mr M Khairuddin, Malaysia
Mr N P Sharma, Nepal
Ms J Paske, New Zealand

Group C Topic 6: Dr D S Naga, Indonesia
Mr P T Wijetunga, Sri Lanka

Topic 7: Dr D K Sinha, India
Dr G C Nguyen, Viet Nam

Problem Solving:
Mr R J Gaffney, Australia
Mr H N Gupta, India
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III Members of the Working Groups for the Preparation of Draft Report

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Mr H N Gupta, India
Mr E Nagasaki, Japan
Mr M P Khairuddin, Malaysia
Mr N P Sharma, Nepal
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Mr S I H Bokhari, Pakistan
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Ms M L Goh, Singapore
Mr P T Wijetunga, Sri Lanka
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Dr G C Nguyen, Viet Nam

Appendix III Responses to the Questionnaire Concerning Mathematics Curriculum

Summary of Answers

Prior to the Workshop, participants were requested to complete the questionnaire concerning elementary mathematics curriculum. In the following, the overall summary of the responses to each of the questions and the overall summary responses to each question are given.

The Latest Revision of Mathematics Curriculum

Question 1: With reference to the latest revision of your curriculum, please indicate the changes occurred in terms of the teaching topics omitted and added in elementary mathematics.

Mathematics curricula in all levels have been changed in response to the social changes and the development of mathematics, science, psychology and education. Some topics have been omitted or added in the revisions made.

Members from eighteen countries participating in this workshop were requested to answer when and how the latest revision was carried out. The year of the latest revision of mathematics curriculum of the elementary level among fifteen countries ranges from 1974 of Fiji to 1984 of New Zealand. There are four countries whose curricula were revised before 1980: Fiji in 1974, 1976, 1983, 1984, Indonesia in 1975, Bangladesh in 1976 and Thailand in 1978. China, Pakistan and Singapore are now revising their curricula.

Of the countries whose curricula were revised in the 1980's, remarkable changes seem to have taken place in Australia, New Zealand, the Republic of Korea and Viet Nam. The number of topics omitted and/or added in these countries is more than five.

The total number of topics omitted in fifteen countries is around forty. Topics omitted can be broadly divided under three headings: Modern Mathematics, the British system of weights and measurement, and complex calculation. These topics are listed with the names of countries in which they were omitted as follows:

1) Modern Mathematics
   Set: India, Japan, New Zealand, Pakistan, Papua New Guinea and the Philippines
   Bases other than ten: New Zealand, Pakistan and Republic of Korea
   Negative numbers: Japan
   Modular arithmetic: New Zealand and Pakistan

Logic: Papua New Guinea
This has been done as a remedy for what was considered as going too far.

2) The British System of Weights and Measurements
Fiji, Malaysia, New Zealand, the Philippines, Sri Lanka and Thailand

3) Complex Calculation
Operations on fractions: Australia and New Zealand
Long division: Australia and New Zealand

The total number of topics added is around thirty, which can be classified under three headings: problem solving, estimation and the metric system. These topics are listed with the names of countries in which they were added.

1) Problem solving
Australia, Pakistan, Papua New Guinea and Republic of Korea

2) Estimation
Estimation: Australia, Indonesia and New Zealand
Rounding: New Zealand and Thailand
One reason for the omission of complex calculations and the inclusion of estimation might be the introduction of calculators into classrooms.

3) Metric system
Bangladesh, Fiji, Malaysia and Sri Lanka

History of Mathematics

Question 2: Do your elementary mathematics textbooks or teachers' guides contain any figures and elements in the history of mathematics? If any, please state them.

Children are likely to think that mathematics is a ready-made subject. But mathematics as a discipline is constantly in the process of development. To make them understand its nature, one effective method is through the activities of problem solving, and the other is through learning something of the history of mathematics. History of mathematics is the history of human ideas. To know how these ideas were born and fostered is useful for the learning of mathematics.

Textbooks/teachers' guides of seven countries contain some references to mathematicians and events in the history of mathematics. They are China, Fiji, India, Japan, New Zealand, the Philippines and Viet Nam.

The following are the names of famous mathematicians quoted in the textbooks followed by the names of countries which published the textbooks:

Gauss: Fiji, Japan and New Zealand
Archimedes: Fiji and New Zealand
Eratosthenes: Fiji and New Zealand
Pythagoras, Quetelet, Thales: Japan and New Zealand

Fibonacci, Napier, Newton, Pascal: New Zealand
Zu Chongzhi: China

The following are topics or historical events quoted in the textbooks and the names of countries which published the textbooks. These topics are divided under two headings: number and numeration, and measurement.

1) Number and Numeration
   Number: Japan and Viet Nam
   Form of notation, methods of computing, place value: —About Egyptian, Babylonian, Spanish, Japanese, Hebrew: New Zealand
   —About Roman: New Zealand and the Philippines
   Greek's multiple of the unit: New Zealand
   Latin's fraction of the unit: New Zealand and Viet Nam
   π's value: China and Japan
   Invention of O: India
   Magic square, multiplication tables: China, Fiji and New Zealand
2) Measurement
   Ancient units of measurement: Fiji and Japan
   SI or metric system: China, Fiji, Japan and New Zealand
   Measurement of height of slope: Japan

Concerning Some Exercises

Question 3: In which school year and at what age of the child does each of the following problems *first appear* in the mathematics curriculum?

The following are the range and mode of school year and of child's age in eighteen countries that each problems first appear in mathematics curriculum.

Problems	School year		Age of child	
	range	mode	range	mode
68+25	1—3	2	6—9	7
804−347	2—4	2, 3	6—10	8
784.92 27.38 63.67 + 597.59	4—6	4	8—11	10
$2\frac{3}{5}+5\frac{7}{12}$	4—7	5	8—12	10
684×342	3—7	4	7—12	9

Problems	School year		Age of child	
	range	mode	range	mode
$2\frac{3}{4} \div 1\frac{5}{8}$	4 — 10	6	8 — 14	11
375.24 divided by 17.3	4 — 7	6	10—12	10—12
Find mentally (use no paper and pencil) 4×239	3 — 7	5	8 — 12	10
What is the number of which 15% = 6 ?	4 — 8	6	8 — 13	11
The multiplication tables up to 10×10	1 — 6	2	6 — 12	8

Some of these problems are not included in elementary mathematics of some countries, and those problems and countries are:

$2\frac{3}{5}+5\frac{7}{12}$: Australia (Victoria) and New Zealand

684×342: Australia (Victoria) and Singapore

$2\frac{3}{4} \div 1\frac{5}{8}$: Australia (Victoria)

375.24 divided by 17.3: Australia (Victoria) and Singapore

Find mentally 4×239: Japan, Pakistan, the Philippines, Singapore and Thailand

It was noted that calculators are being used in Australia and New Zealand for the following computation:

684×342: New Zealand

375.24 divided by 17.3: Australia (Tasmania) and New Zealand

Iufluence of Modern Mathematics

Question 4: Do you see any impact or influence of "modern mathematics" in current syllabus or course of study of elementary mathematics?

"Modern Mathematics" or "New Mathematics" movements have enormously affected mathematics education during the past decade all over the world. To improve mathematics education, evaluation must be made on the impact or influence of these movements on current mathematics curriculum. The impact or influence of "modern mathematics" on the current syllabus or the course of study in elementary schools in eighteen countries can be classified into three components, namely, objectives, content and methods.

The impact of "modern mathematics" on objectives is different country by country. Examples are the emphasis on understanding of the mathematical concepts and principles in Indonesia, Pakistan, the Philippines and Thailand; the emphasis on logical and psychological ways of thinking in Nepal; and the emphasis on ability in problem solving in Fiji, New Zealand and the Philippines.

The influence of "modern mathematics" on contents can be seen through the inclusion of sets, laws of number systems, probability and intuitive geometry in the curriculum. Some of them are taught through concrete examples as a germ of idea. The following are some contents mentioned by more than one country.

Sets: Australia, Bangladesh, China, Fiji, Indonesia, New Zealand, Pakistan, Republic of Korea, Sri Lanka and Viet Nam

Laws of number system: Japan, Australia, Fiji, Indonesia, New Zealand, Thailand and Viet Nam

Number patterns, relation and simple graph: Fiji, Indonesia, New Zealand, Pakistan and Sri Lanka

Probability: Indonesia, Japan and Republic of Korea

Statistics: Fiji, New Zealand and Republic of Korea

Ordered pairs, symmetry, congruence: Fiji, Indonesia and Thailand

Function: China and Pakistan

Other contents mentioned by one country are as follows:

Variables: Japan

Number axis: Viet Nam

In addition, New Zealand mentioned that the content has been refined to meet "the needs of today's children". Regarding the influence of "modern mathematics" on teaching/learning methods, a teacher-centred teaching style, and rote type of learning by children have been deemphasized, instead, a discovery approach, conceptual approach and learning through understanding have been emhasized. The following are these types of methods and name of countries using them.

Discovery approach: New Zealand, Pakistan and the Philippines

Conceptual approach: Indonesia, Nepal and Pakistan

Learning mathematics through understanding: Fiji, Malaysia, New Zealand, Singapore and Thailand

Activities and experiments: Fiji, Indonesia and New Zealand

Problem solving in New Zealand and the Philippines, the use of environmental materials in India and concrete aids in Malaysia are other influences on method. Pakistan has provided low cost teaching kit to every elementary school for improving classroom teaching.

Two other influences were mentioned: Difficulties in obtaining competent teachers and parents help for their children (Indonesia); and economic development (Papua New Guinea).

Country-wise Answers

The Latest Revision of Mathematics Curriculum

Question 1: With reference to the latest revision of your curriculum, please indicate the changes occurred in terms of the teaching topics omitted and added in elementary/primary mathematics.

Australia

Topics Omitted in the Curriculum Revision of 1982-84 in Victoria State:

Vulgar fractions operations; extended notation for arithmetical processes; problems such as $\frac{2}{3}\times\frac{5}{6}-\frac{4}{3}$ of 5; long division with two digit divisor (will be done with calculator); and long multiplication with 3 digit multiplier

Topics Added in 1982-84 in Victoria State:

Mental computation; estimation with operation with number; use of the calculator; and problem solving

Topics Omitted in the Curriculum Revision of 1978 in Tasmania State:

Arithmetic in base other than 10; modular arithmetic; and set theory

Topics Added in 1978 in Tasmania State:

No significant additions of content, but more attention to how learning was effected.

Bangladesh

Topics Omitted in the Curriculum Revision of 1976:

None.

Topics Added in 1976:

Basic concept of set and metric system of measurement: cm/m and g/l

China

None.

Fiji

Topics Omitted in the Curriculum Revision of 1975 for Classes 1 and 2:

Imperial measure: ft/in, ft/yd, pint/gallons

Topics Added in 1975 for Classes 1 and 2:

Metric system of measuremnt: cm/m, g/l

Topics Omitted and Added in the Curriculum Revision of 1974 for Classes 3 to 6:

No topics omitted and added, but only simplified.

Topics Omitted in the Curriculum Revision of 1984 for Class 7 and 1983 for Class 8:

No topics omitted for class 7 in 1984, but difficult exercises were omitted and replaced by simpler ones for class 8 in 1983.

Topics Added for Class 7 in 1984 and for Class 8 in 1983:

No new inclusion, but new exercises and more diagrams are added on same topics.

India

Topics Omitted in the Curriculum Revision of 1983:

Concept of a set and operations on set

Topics Added in 1983:

Graphs: Bar graphs, their construction and use (with actual mathematics data for comparison purpose)

Indonesia

Topics Omitted in the Curriculum Revision of 1975:

Composite computations

Topics Added in 1975:

Sets; Inequalities; negative numbers; algebraic expression; coordinate system; geometry; estimation; probability; statistics; and nondecimal number system

Japan

Topics Omitted in the Curriculum Revision of 1980:

Terms and symbols of the set (grade 4); property of equations (grade 4); and negative numbers

Topics Added in 1980:

None.

Malaysia

Topics Omitted in the Curriculum Revision of 1981:

Imperial system in measurement; square and square roots of numbers; cube and cube roots of numbers; geometry—angles, lines, symmetry, position and rotation of coordinate, use of geometrical instruments; and introduction of algebra

Topics Added in 1981:

Metric system in measurement

Nepal

Topics Omitted in the Curriculum Revision of 1979:

Place value (grade 1); sign of comparison (grade 2); number sentence (grade 2); simplification of two digits using only plus (+) and minus (−) (grade 4); and use of calendar

Topics Added in 1979:

Place value (grade 4); missing adding (grade 3); addition and subtration of like fraction (grade 3); and simple problem relating to daily life on unit and measurement

New Zealand

Topics Omitted in the Curriculum Revision of 1984 for Grades 1 to 5 (Age 5 to 10 years):

Formal notation about sets; cross products; use of large numbers in

grades 3 to 4; study of bases other than 10; study of historical number systems; exponents; long division; modular arithmetic; formal logic; fractional numbers as a separate section; +/− of mixed numbers; ×/÷ of fractions by fractions; ratios; deriving sets of ordered pairs; primes; multiples factors; common factors, etc.; premature introduction use of precise terminology; notation in geometry such as ←→ ⟶ m (AB) etc; finding mean, median and range; equal adjustments method of subtraction; and open and closed sentences

Topics Added in 1984 for Grades 1 to 5 (Age 5 to 10 years):

Rounding; estimation before calculation; decomposition method of subtraction; and "sharing" aspect of division

Topics Omitted in the Curriculum Revision of 1984 for Grades 6 to 10 (Age 11 to 15 years):

Long division; +/− mixed numbers; matrices; use of slide rule; logarithm; irrational numbers; and group theory

Topics Added in 1984 for Grades 6 to 10 (Age 11 to 15 years):

Estimating; use of calculator; budgeting and other applications to everyday situations

Pakistan

Topics Omitted in the Curriculum Revision of 1985 for Grades 1 and 2, and in 1986 for Grades 3 to 5:

Sets and operations

Topics Added in 1985 for Grades 1 and 2, and in 1986 for Grades 3 to 5:

Problem solving

Papua New Guinea

Topics Omitted in the Curriculum Revision of 1977 to 1982:

Sets; logic; and bases other than 10

Topics Added in 1977 to 1982:

Problem solving; and Money

Philippines

Topics Omitted in the Curriculum Revision of 1982:

Set theory; set notation; operation with sets; bases other than 10; and British system of weight and measurements

Topics Added in 1982:

None

Republic of Korea

Topics Omitted in the Curriculum Revision of 1992:

Base 2 notation; base 5 notation; multiplication and division with time; relative error; exponents; congruent numbers (expression) and their properties; negative rational numbers; and concept of topological geometry

Topics Added in 1982:

Problem solving

Singapore

None.

Sri Lanka

Topics Omitted in the Curriculum Revision of 1981 for Grades 2 to 5:

British measuring units

Topics Added in 1981 for Grades 2 to 5:

Metric measuring units; and international standard practice of writing numbers (scientific and technical purpose)

Topics Added in 1984 for Kindergarten and Grade 1:

Recognizing, reading and writing number from 1 to 6 extended to 9; mathematical concepts—numbers 6, 7, 8 and 9, addition: total up to 5; comparing and ordering numbers from 1 to 6 extended to 9; ordinal numbers from 1 to 6 extended to 9; and addition and subtraction money upto Rs 100.

Topics Added in 1984 for Grade 2:

Place value extended up to 999

Topics Added in 1984 for Grade 3:

Introducing and using all coins and currency notes; addition and subtraction of hours and minutes; mapping one to many (1:2, 1:4, 1:10); number and place value extended up to 9999; roman numbers from I to X; number patterns odd and even numbers; addition total not exceeding 9 999 (3 lines with and without carrying over); solution of problems involoving addition and subtraction connected with the day-to-day life (number not exceeding 99); solution of problems: problems involving addition, subtraction and multiplication—not exceeding two operations at a time (number not greater than 999)

Topics Added in 1984 for Grade 4:

Addition and subtraction; money extended up to Rs 200 by transaction; solution of problems: problem involving addition, subtraction and multiplication and division—not exceeding two operations at a time (numbers not greater than 9 999).

Thailand

Topics Omitted in the Curriculum Revision of 1978:

British measurement system; using the protractor to measure length of the second place decimal of centimeter; conversion of foreign money to Thai money; drawing pie chart; proportion; statement of charges, receipt, and cash account of shop keepers

Topics Added in 1978:

Symmetry; congruence; graph of ordered pairs; equation; rounding, and personal balance

Viet Nam

Topics Omitted in the Curriculum Revision of 1980:

Solving some types of problem such as problems concerning two moving objects on a straight way, problems concerning planting (along a way or around a rectangle garden)

Topics Added in 1980:

Notion of relations of quantities (equality, inequality) in grade 1; simple equations relative to the addition and the subtraction in grade 2; simple equations relative to the multiplication and the division in grade 3; literal expressions and simple inequalities in grade 4; and law of association, law of communication of the addition, multiplication operations

History of Mathematics

Question 2: Do your elementary mathematics textbooks or teachers' guides contain any figures and elements in the history of mathematics? If any, please state them.

Australia

No direct reference to these.

Bangladesh

The elementary textbooks on Mathematics contain many figures and elements to illustrate the subject matter and to make the topics easier for the learners. But those elements or figures are not directly related with the history of Mathematics.

China

In our textbooks we show that more than 1400 year ago Zu Chongzhi, a mathematician in our ancient times, found π's precise value is between 3.1415926 and 3.145927.

In teachers' guides we have briefly introduced: (1) the history of the International System of Units; (2) the study of π's value, magic squares and multiplication tables in verse in our ancient times.

Fiji

Very brief mention of the elements in the history of mathematics. Those included are: (1) some mention of ancient units of measurement in the unit 'measurement'; (2) Gauss: index notation when teaching indices; (3) Eratosthenes—prime numbers; and (4) some mention of Greek mathematicians Archimedes in the calculation of earth's circumference (topic 'circles')

India

Books written on Arithmetic by Late Gopal Krishna Gokhale contains historical note at the end of chapters. This was written in 1896. In some

state-textbooks, some reference is given to '0'. This was invented by Hindus.

Indonesia

No.

Japan

Three series among six authorized textboox series contain the stories as follows: illustarated history of numbers (Grades 4 and 6); history of π (Grade 5); origins of measuring units (Grade 5); history of metric system (Grade 6); Measure of height in Edo era (Grade 6); history of measurement (Grade 6); and the episodes of Gauss, Pythagoras Quetelet and Thales (Grade 6).

Malaysia

No.

Nepal

No.

New Zealand

Only M.S.M. Books 6 and 7 (Grades 6 and 7). The following persons are briefly referred to: Fibonacci, Napier, Gauss, Newton, Archimedes, Pascal. The topic "Numeration" provides references to Egyptian, Babylonian, Spanish, Japanese, Roman, and Hebrew forms of rotation or methods of computing and in the Measurement section there is some reference to prefixes used i.e. Greek for multiples of the unit and Latin for fractions of the unit.

Pakistan

No.

Papua New Guinea

No.

Philippines

There is a lesson on how early man did his counting and how ancient Romans read and wrote their numerals to enable he learners to see the advantages of place value numeration over othe system. Objects are counted by matching each with a counter. Counters naybe stones, knots on a rope, notches on a tree or tally marks.

Republic of Korea

No.

Singapore

No.

Sri Lanka

No.

Thailand

No.

Viet Nam

The numbers first used by man were the natural numbers; a number is an idea that is associated with a group of things (mathematics is not divorced from real life).

Fractions were known to the Egyptian as early as approximately 1650 BC, but it was the Arabs who introduced the modern method of writing fractions and this method was generally in use after approximately 1140 AD.

Concerning Some Exercises

Question 3. In which school year and at what age of the child does each of the following problems *first appear* in the mathematics curriculum ? [This question is partially adopted, with the purpose of international comparison, from the Comparative Studies of Mathematics Curricula conducted by the Institute for the Didactics of Mathematics (IDM) in 1980.]

	$68 \div 25$		$804 - 347$		$\begin{array}{r} 784.92 \\ 27.38 \\ 63.67 \\ +597.59 \end{array}$		$2\frac{3}{5} \div 5\frac{7}{12}$		684×342	
	Grade	Age	Grade	Age	Grade	Age	Grade	Age	Grade	Age
Australia	V* T** 3 3	V T 8 8	V T 4 4	V T 9 9	V T 5 6	V T 10 11	V T — 7	V T — 12	V T — 6 (With calculator)	V T — 11
Bangladesh	1	6	2	7	4	9	4	9	3	8
China	1	7	2	8	4-5	10-11	4-5	10-11	3-4	9-10
Fiji	3	8*	3	8*	5	10*	5	10*	6	2*
India	2	6*	2	6*	5	9*	4	8*	4	8*
Indonesia	2	8	3	9	4	10	5	11	4	10
Japan	2	7	2	7	4	9	5	10	4	9
Malaysia	2 -	7*	3 -	8*	5 -	10*	7 -	12*	7 -	12*
Nepal	2	7	3	8	5	10	5	10	4	9
New Zealand	3	7*	4	8	6	10	—	—	7 (with calculator)	12
Pakistan	2	6*	2	6*	4	8*	4	8*	3	7*
Papua New Guinea	3	9	4	10	4	10	5	11	6	12
Philippines	1	7	2	8	4	10	6	12	5	11
Republic of Korea	2	7	3	8	4	9	5	10	4	9
Singapore	2	7	2	7	4	9	5	10	—	—
Sri Lanka	1	6	3	8	4	9	5	10	4	9
Thailand	2	7	4	9	5	10	6	11	4	9
Viet Nam	2	7	3	9	5	10	6	11	4	9

* Victoria Stage ** Tasmania State

	$2\frac{3}{4} \div 1\frac{5}{8}$		375.24 divided by 17.3		Find mentally (use no paper and pencil) 4×239		What is the number of which 15% = 6 ?		The multiplication table up to 10×10	
	Grade	Age	Grade	Age	Grade	Age	Grade	Age	Grade	Age
Australia	V T — 7	V T — 12	V T — 7 (With calculator)	V T — 12	V T 5 5	V T 10 10	V T 8 7	V T 13 12	V T 4 4	V T 9 9
Bangladesh	5	10	5	10	5	10	5	10	2	7
China	5 - 6	11 - 12	4 - 5	10 - 11	3 - 4	9 - 10	5 - 6	11 - 12	1 - 2	7 - 8
Fiji	7	12*	7	12*	5	10*	7	12*	4	9*
India	5	9*	6	10*	5	9*	6	10*	2	6*
Indonesia	5	11	6	12	4	10	5	11	2* and 1	6* and 9
Japan	6	11	5	10	—	—	5	10	3	8
Malaysia	7 -	12*	7 -	12*	7 -	12*	7 -	12*	3 -	8*
Nepal	4	9	6	11	5	10	6	11	2	7
New Zealand	9	14	8 (with calculator)	12	7	11	7	11	4	8
Pakistan	4	8*	6	10*	—	—	4	8*	2	6*
Papua New Guinea	5	11	5	11	5	11	5	11	4	10
Philippines	6	12	6	12	—	—	6	12	3	9
Republic of Korea	6	11	6	11	3	8	6	11	2	7
Singapore	6	11	—	—	—	—	6	11	3	8
Sri Lanka	5	10	6	11	6	11	7	12	5	10
Thailand	6	11	6	11	—	—	7	12	2	7
Viet Nam	6	11	5	10	4	9	6	11	6	11

Influence of Modern Mathematics

Question 4: Do you see any impact or influence of "modern mathematics" in current syllabus or course of study of elementary mathematics? If so, please describe them briefly.

Australia

(Victoria) As exemplified by sets, axiomatics—no.

(Tasmania) Notion of a set is used. The vocabulary of sets is developed to provide precision in the use of language. Little or no reference to "laws" such as "the distributive law" but it is met through examples.

Bangladesh

Present curriculum of elementary mathematics has been developed to fulfil the demand of "modern mathematics" of secondary and higher secondary levels. In the current syllabus of elementary mathematics there

is scope and opportunity for the students to acquire the basic concepts of "set" which play a most important part in "modern mathematics". Our previous syllabus was devoid of the topic, and there is impact of modern mathematics in the current syllabus.

China

In elementary school mathematics we use teaching aids to enable children to have some contact with some new ideas in "modern mathematics" such as sets and function, etc. We use these new ideas to help children to deepen their understanding of certain content such as the meaning of the four operations, H.C.F. and L.C.M., relations of geometric figures, etc. But the definition of and symbols for these ideas are not introduced.

Fiji

Not in particular. Our new maths programme has the blend of old and new courses. We do not teach the number system other than base 10. There is a lot of change in teaching styles. Previously, the teaching was mostly teacher centred. Now, there is a lot of emphasis on activities/experiments, etc.

India

Cramming by children is reduced. More environmental material is used.

Indonesia

Modern mathematics replaces arithmetic in the curriculum of elementary education. Hence, there is a great change in the syllabus. This change mainly pertains to the introduction of concepts rather than to the skill of computation. It is intended to promote cognition and thinking abilities as well as to provide a means for coping with the tremendous progress in science and technology.

Japan

According to the latest revision of the course of study (1980), the symbolic representation on sets, probability are explicitly omitted. However, the notion of probability has remainded, and the idea of variables and the property of operation are firmly placed in the course of study.

Malaysia

Contentwise, there is not much difference between the present and the previous mathematics curriculum. However the instructional approach for the topics now stresses the use of concrete aids and items to facilitate understanding or meaningful learning, followed by drill exercises. This

increased emphasis on learning mathematics through understanding is very much due to "modern mathematics."

Nepal

With the advent of national education system plan (1971) mathematics has been given a significant place at primary level. Of the total time for instruction, 30% is alloted to mathematics at primary level. Mathematics is given a very important place second to language. The Primary Math Curriculum is based upon a conceptual approach and it has also tried to develop logical and psychological ways of thinking, understanding, learning and working on simple mathematical problems. Thus, to some extent, there has been some impact of modern mathematics.

New Zealand

Modern mathematics has influenced mathematics education in the primary school in New Zealand since 1967. Since then the content of the syllabus has been refined to better meet the needs of today's children but the teaching approach remain based on understanding with realistic specific objectives being set. Any topic is introduced with a problem worth solving, materials are used to solve the problem, children are encouraged to suggest suitable written forms, these are refined and a standard form practiced. The questions asked by writers of syllabuses are "Is it relevant?" "Is it realistic?" "Is it useful for later learning?".

Pakistan

The idea of set and operations on set were taught in primary classes. This topic will be deleted from the year 1985. The following changes however took place in curriculum and teaching under the influence of modern mathematics.

Objectives: The curricular objectives now emphasize understanding of concepts rather than mastering of repetitive skills.

Contents: The following topics have become a part of elementary mathematics curriculum under the influence of modern mathematics: i) One-one correspondence, ii) Number ray and number line, iii) Open and close figures, iv) Commutative and associative properties of operations.

Teaching: Teaching is now concept oriented. More emphasis is given to the use of instructional aids.

Papua New Guinea

I have seen tremendous impact of "modern mathematics" in the current syllabus for elementary mathematics, especially in the lower level of economic development, and in the areas of rural type of marketing and

trading.

Philippines

The modern mathematics has an influence in the methodology of teaching elementary mathematics. Rote-type of learning no longer has a place in the curriculum. The discovery approach is emphasized to enable the learners to understand the mathematical concepts and principles. Problem solving is underscored to enable the learners to see the uses and application of mathematics in daily life.

Republic of Korea

Elementary school mathematics includes basic concepts of set theory, probability and statistics.

Singapore

No.

Sri Lanka

Ideas of sets; Relations; Number patterns; and Simple graph.

Thailand

The areas of impact are: Emphasis on understanding the concept; New contents such as symmetry, congruence, order pairs; Some contents introduced in the lower grades.

Viet Nam

The current syllabus of primary mathematics does not contain any topic of modern mathematics, but the idea of modern mathematics permeate the course of study.

The beginnings of arithmetic are initiated with sets of objects. Children study sets of different sizes. They learn to abstract the number property of the set and associate a fixed number name to all sets of the same size.

Addition is interpreted as finding the number of the combined set from the numbers of the separate sets. The intuitive use of commutative and associative laws plays a large role. Subtraction is related to finding the complement of set or the set of objects remaining. The operations on physical sets enable children to see how addition and subtraction of numbers are reversible operations. Multiplication is looked upon as explaining the result of replacing each element of a given set by other sets, each set having the same number. The intuitive use of the commutative and distributive laws plays a large role. Division is seen as the reverse of multiplication, that is given a set of objects, partition this set into a

number of equal subsets. If the size of the subset is known, we are asked to find the number of these subsets, and if the number of desired subsets is known, we are asked to find the size of each subset. The couse of study in elementary mathematics gradually acquaints children with the idea of number axis.

Appendix IV Manipulative Materials as an Aid to Learning Mathematical Structure and Problem Solving*

Our tasks for this Workshop are clear: to examine the role played by teaching materials in learning mathematics in the elementary school, and to give examples of some of these materials and their use. Let us look briefly at why mathematics is taught in our schools. Why are more hours given to mathematics in curriculum than to any subject other than language? The answer is simply that mathematics is so useful. Much of the current criticism of mathematics teaching is that it is being taught without life problem solving skills. But we cannot possibly know which notions and skills today's elementary school children will need as adults, especially concerning computational skills in an era of technological developments. Thus, our concern must be with the mental development of the child, to foster the ability to view a given situation in mathematical terms, and to teach some skills to transfer knowledge.

You are all aware of lists of objectives in teaching mathematics. Let me report on a similar workshop, the Council of Europe Workshop on Renewal of Mathematics Teaching in the Primary School held in Switzerland in May this year. The participants agreed that in defining aims for future innovations, concentration should be given more on the adaptability of these aims to the level of development of the pupils, to their level of language, and to personal experience rather than their fulfilment of external demands such as the structural concerns of mathematics, the pressure of potential users, or the fashions and trends of the moment.

This had been stated in a different way at a Unesco international meeting on the goals of mathematics education, in a report that was published as *Studies in Mathematics Education*, volume 2. That meeting recommended that whenever possible mathematics be taught through situations that exist in society, and that pupils learn to recognize and formulate the mathematics existing in those situations in order to develop practical problem-solving skills. Through a carefully planned sequence of situations, the pupils will gradually develop cognitive structures of mathematics and learn how these situations actually exist in their world. This approach should be a goal to be reached, in a gradual evolution of shifting from the type of mathematics teaching that now takes place in most of our schools. Help must be given now to equip schools with materials for a more active approach to teaching mathematics, and constant encouragement given to teachers to examine local situations for their

* This paper was submitted by Ed. Jacobsen from Unesco, Paris who participated in the workshop.

mathematical content and utilize these situations in their teaching. Teacher's guides certainly are needed for this. These points in more detail are discussed in my article that just appeared in the Unesco Regional Office for Education in Africa's quarterly bulletin, *Educafrica*.

Two implications for this Workshop arise from the preceding remarks:

- we must develop teaching/learning materials that will assist in promoting a more active approach in mathematics classes;
- we must think of real situations familiar to our pupils to evoke the mathematics we think important for them to learn, and which may invoke problem solving skills.

Since our major activity will be to develop manipulative mathematics materials, let me report on what research has shown about their effect on pupil learning. I apologize for quoting research from a different region, but I am not familiar with any in this field from Asia. ERIC in the United States of America summarized hundreds of research projects on manipulative materials. They found that using concrete objects or seeing them demonstrated improved their pupils learning of concepts. As children work with objects and talk about what they are doing, meaning is created and assimilated. This activity should precede drill and practice, which should precede practice activities with symbols. In the USA, teachers make wide use of concrete materials in the early years of schooling, but less frequently in later grades. What can research tell us about the continued use of concrete objects in mathematics learning? Half of the studies showed that pupils using manipulative materials score significantly higher on achievement tests than those who have not used them. The other half of the studies showed that they scored the same (but the tests were largely 'doing sums'). The most effective sequencing of activities was first to use concrete objects, then pictorial representation of these objects, and lastly the symbolic representations.

What about using a variety of concrete objects, such as the abacus, blocks and place value charts to represent one mathematical idea? Research shows little difference between using several objects or only one. It is difficult for me to accept this, since concept learning requires both examples and counter-examples of the concept in order to indicate boundaries and, seemingly, a variety of examples, rather than just one, would accomplish this better. Most tests do not measure concept learning, I argue, as I look for a way to discount research that does not fit what I want to believe!

Another resaerch result showed that it made little difference whether the materials were teacher-demonstrated or used by the pupils themselves. Research also indicates that using manipulative materials is as effective in the higher grade levels as in the lower. And are they of benefit only to slow learners? No; again research shows that pupils with high ability profit as much as those with low ability, although the high ability pupils do not require their use for as

long a period. Continued use delays the pupils from developing algorithms.

In summary, research in the USA has shown that by utilizing manipulative materials, all children established better cognitive structures. Many manipulative materials are highly structured, and may lead to an emphasis on structural, content-oriented aspects of mathematics in elementary school, rather than on aspects more directly related to the natural sphere of the child. The personal experiences of the child should be more important than the external demands of the structure of mathematics. Therefore, in addition to selecting artificial aids such as place-value charts, blocks, pie charts to develop mathematical concepts, we must consider materials that relate to the real world. Every community abounds in materials with a mathematical basis. After all, mathematics *is* useful. Such materials help children make the link between mathematics as theory and mathematics as a useful tool. For a discussion on developing a curriculum based on learning experiences that emphasize the uses of mathematics, it is recommended to refer to Max Bell's article, "Teaching Mathematics as a Tool for Problem Solving", that appeared in Unesco's *Prospects* (Vol. IX, No. 3, 1979). Bell lists topics that are useful to most people, organized in a way that reflects the mathematical modelling process, i.e., "skills related to abstraction and symbolization are listed first, then, skills primarily needed in doing mathematics, then, skills needed in using mathematical information from whatever source in interpretation, prediction or decision-making" (Bell, p. 315). Let us find, during this workshop, a balance between the two types of materials for teaching mathematics.

Appendix V Expected Function and Role of Instructional Materials in Elementary Mathematics Education (Summary)*

Introduction

A fairly banal view about instructional materials in mathematics is that these are materials intended to cater to instructional processes, whatever be the level of mathematical education. While the content and methodology of such materials obviously ought to vary from level to level, their complexion has changed over the years. A deeper glance over these materials reveals a shift in perspectives in recent times, that can only be brought to the fore by reckoning what are going on in the international setting. Instructional materrials on primary mathematics have distinctively different overtones being increasingly realized and this is what has also been doctored out in this paper. All these taken in totality are sought here to focus not merely what ought to be envisaged and are expected for such materials, but also the strategies and modalities, necessary in order to translate them into useful and meaningful realities.

Some Preliminary Remarks about Primary Mathematical Education vis-a-vis Instructional Materials

A view that has come to stay is that instruction in any subject, and certainly in mathematics, at the primary (or elementary) level, can hardly be delinked from instructional processes in other subjects at the same level. Indeed, the view that subject-wise instructional components need to be interwoven in a studied manner has become predominantly discernible over the years. "Activity" has been and continues to be the keyword in any kind of mathematical instruction at the primary level. "Activity" in mathematical instruction takes cue from the totality of concrete experiences, past or present. The transfer of concept(s) from extra-mathematical reality to those in mathematical reality is the ultimate objective. No efforts are minced for improvization so that mathematical concepts take roots in the conception by pupils. By "instructional materials", one could have "textual materials" only or strictly speaking, "texts". Not a shared of "activity" of the present brand could find place of traditional types in mathematical texts. There used to be problem-sheets but those, again, with the tenor, that did hardly smack of

* This paper was submitted by D. K. Sinha from India, who participated as a resource person in the workshop.

modernity, whatsoever. That something over and above the textual materials being required is the idea of fairly recent origin that has grown out of fundamental thinking on primary education and more so, on mathematical education at the primary level. This has vastly changed the pattern of primary school education. Research in children's method of learning has contributed considerably to the new scenario on primary school education, which has well facilitated the shift in emphasis from mathematics teaching to mathematics learning. Identification of age-groups for purposes of learning has considerably helped in categorization of mathematical topics for purposes of instruction. "Active learning" in mathematics is being harped upon at the primary level. Instructional materials in primary mathematics, in principle, ought to reckon changing accent in respect of mathematical education at the primary level. They range from textual materials, work (or problem) sheets, assignment cards, charts, models, picture cards, etc. to recreational, enrichment, supplementary materials—all designed for pupils.

International Perspectives

Of all activities in the international arena, those with a remarkable conspicuty are the materials prepared under the aegis of UNESCO. The recent publication, Studies in Mathematics Education Vol. 3, even though not directly connected with instructional materials, provides in an ample measure ideas about what such materials ought to be. In fact, the articles in this volume are dominantly teacher-education-oriented. In lead articles, Fonacier and Oteiza plead for an environmental bias to the mathematical curriculum at the primary level and these cannot but find consideration in the preparation of instructional materials. There are pedagogical issues, case studies, subject-wise difficulties (particularly on geometry), etc. that ought to be assiduously kept in view if worthwhile instructional materials are to be prepared. What makes this volume singularly distinctive is to deal with the role of instructional materials essentially for purposes of in-service programmes of teachers (without degrees). Some essential features of this case study ought to be studied in some volumes of New Trends in Mathematics Teaching published few years ago by UNESCO also dealt with partly aspects of primary mathematics.

The appropriate deliberations in International Congress on Mathematical Education (ICME) right from the Exeter Congress to the Adelaide Congress have brought to the fore problems, trends and needs of mathematical education at the primary level. A tremendous spate of interest and concern is reflected by areas deliberated during the last fifth ICME held in August 1984 at Adelaide, Australia. Some of these areas were deliberated in two Action Groups, dealing specifically with early childhood years (ages 4 - 8) and elementary school (ages 7 - 12) respectively. There was hardly any Theme Group which did not address

to itself at least one topic pertaining to primary mathematics. There was, indeed, a plethora of papers presented orally and/or through posters, on primary mathematics in this Congress. One ought to refer to an another important conference, namely, eighth International Conference for Psychology of Mathematics Education at Sydney, held just few days before 5th ICME, in which very important and substantive deliberations took place on mathematics being doled out at the primary level, of course, their psychological implications.

Deliberations in regional, binational and similar get-togethers ought to wield influences in shaping our perspectives. For examples, APEID report of Study Group. The proceedings of Anglo-Soviet Seminar are useful publications in this direction. The famous Cockroft's report of U. K. has many things to say about primary mathematics. The report of previous mathematics workshop organized jointly by NIER-ACEID is perhaps the most recent one and ought to be treated, for many reasons, as a vademecum for setting out apposite perspectives on primary mathematics, without being obligious of trends of thinking and activities at the international level.

Formulation of Ideas about Instructional Materials

A total view of what are set forth in the preceding lines ought to enable us to carve out the form and shape of instructional materials at this primary level. That it should necessarily be of a different genre is obvious. That its texture and format would be unlike what it had hitherto been is well understood. What should, then, be guiding principles in the preparation of such stuff ? A necessary prerequisite before such principles are formulated is that we need to have moorings that have been built up over the years. The "New Maths" movement has suffered a setback and we seem to have swung back to "basics" in mathematics in every sector of mathematical education. It appears that there is an earnest yearning to strike a middle path. The unique feature that pervades through all forms of materials, save those that are avowedly traditional, is that we should cash in experiences of the children, and in fact, the totality of experiences, be it in classrooms, in playgrounds, or in homes. But reality is that mathematical instruction has been dominantly classroom oriented, sometimes to such an extent, that these often stand dubbed as too esoteric and delinked from life experiences. The perennial snag in mathematical instruction is the intrinsic character of mathematics; a purist in his approach to mathematics continues to look askance at improvization made for and in the name of mathematics; the extreme is awful dilution of mathematics. Hence, the need of the middle path again. One cannot abjure 'activities' based on 'experiences' which, in turn, need to be widened. One can do so by providing an environmental tilt to the gamut of experiences. It is the environmental bias that facilitates mathematization of the environment. Modelling of environmental situations,

going back and forther between mathematics and extra-mathematical situations is synonymous with mathematization *per se*. The immediate advantage that one could possibly accrue from this exercise is that occasional dropouts, at this level, so common in agricultural setting of developing countries may get stimulus from reality. Hence, any non-formal counterpart of formal elementary education will be seldom out of unison with each other. The links between mathematics and other disciplines cannot but be more strengthened than ever before. Applications of mathematics would acquire a newer dimension. Employers' points of view do provide often inputs for building up instructional materials.

Strategies and Modalities

Having set forth the guidelines, we need to have the possible layout of instructional materials. One has to exercise finesse and dexterity in the preparation of pupils' textual materials. If mathematics is to be extracted out of environmental stuff, the sequencing of the entire text should move in a spiral fashion; environmental reality provides the motivational input, helps in initiating minimum mathematical element, turns to environmental realities and exploits the entire equipment in furthering mathematical literacy and numeracy and thus goes on the process. Work sheets for pupils may be another form of instructional materials. Mathematical content has to be dinned into the ears and has to be harped on so that mathematical skills are well mastered and problem-solving is well acquired.

Handbooks/guidebooks for teachers are musts in this regard. Each topic is to have an introduction, method of introducing topics, activities (in the form of charts, models, etc.) and mathematical concepts going a bit beyond what is normally required of pupils. Listing of vocabulary and symbols ought to figure in such handbooks for teachers and more so, on account of the growing importance of language and its interaction with mathematics education.

There should be a different brand of instructional materials for mathematics education at the level. Methodology should certainly be there but not at the cost of subject matter.

A possible setback in innovation at the primary level, in this context, is resentment among parents of pupils. This has to be obviated through organization of programmes and preparation of instructional materials of different complexion, specifically meant for them.

These instructional materials cannot be prepared under the aegis of teachers of mathematics only. Environment being essentially inter-disciplinary, the stuff of the kind described above can be prepared only through interaction of teachers of mathematics, mathematics educators and environmentalists of different types. A pilot scheme or an experimental try-out will provide the

feedback and this will enable later on in preparing instructional materials on a massive scale. Supplementary and enrichment materials for better ability of children and reading materials for teachers should also be developed.

Conclusive Remarks

In the foregoing lines, instructional materials of written forms are considered. There exist a host of so-called extra-instructional materials that facilitate learning of mathematics at the primary level. These are often referred to as resources of learning of mathematics. A system approach is yet to be undertaken so as to have a better understanding about the totality of experiences to be capitalized. Three emerging facets of mathematical education, namely, language and interaction with mathematical education, mathematical modelling (including problem solving) and discrete mathematics (including informatics, computers, etc.) need in-depth study and research, in the context of primary mathematics.

Summing up, one can say instructional materials are to be used as catalysts for 'learning of mathematics' rather than 'teaching of mathematics'. In order that these prove to be helpers rather than hindrances to this end, it has to be borne in mind that the teacher is the most important resource for learning of mathematics. Instructional materials in mathematics if they are to be in keeping with trends of the day, ought to be cast in the mould of environment, either natural or man-made, and mathematical instruction will, hopefully, bring about sense of oneness with, and of belonging to the environment. Let us reiterate that instructional materials at this level have a wider connotation and, content too. It is the art of using them with care, finesse and judgment from a holistic point of view that can go a long way in shedding fear for mathematics and can create an abiding interest and love for mathematics. What else, in the ultimate analysis, can be the functions and role expected and envisaged, of instructional materials?